IMPACTS OF CHINA'S CREDIT POLICY ON THE SOE-LED AND INVESTMENT-LED ECONOMIES

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VERY PRELIMINARY AND INCOMPLETE

Abstract. We build one coherent framework to account for the regime change from the SOE-led economy before 1998 to the investment-led economy since 1998. We argue that a switch of credit policy in favor of SOEs to credit policy in favor of capital-intensive industries is the driving force of the observed structural change in China's aggregate data.

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I. Introduction

The share of SOEs in aggregate investment declined steadily during the 1998-2016 period of what Chang, Chen, Waggoner, and Zha (2016) call "the investment-driven economy." But the SOE share has risen rapidly from 20% to 70% in the recent years (Figure 1). Although the number of these years is too short for us to analyze the cause and effect of the rising SOE share in recent history, we can learn from the 1978-1997 period when China experienced a high level of the SOE share. We call the economy during this period "the SOE-led economy." In this paper we offer this historical analysis by first documenting the stylized facts for the SOE-led economy and then providing a theoretical explanation of these facts.

In contrast to the investment-driven economy in which capital accumulation is a driving force of GDP growth, we find that TFP growth was a main driver of economic growth. In the investment-driven economy, it is physical capital that was reallocated from the light sector to the heavy sector. In the SOE-led economy, it is labor that was reallocated from households to firms in the light sector. As a result, the trend and cycle observed for the SOE-led economy are qualitatively different from those observed in the investment-driven economy.

II. Stylized Facts

II.1. **Institutional Background.**

II.2. **Empirical facts.** The trends and cycles in the SOE-led economy are summarized as follows.

- Trends:

(T1) Stationary investment-output ratio (Figure 2).

(T2) Stable labor share of income.

(T3) High shares of SOEs in FAI and in bank loans to investment (Figures 3 and 4).

(T4) Stable ratio of gross output (measured by the ratio of sales revenues) and capital stock (measured by gross fixed asset) in the heavy sector to that in the light sector (Figures 5 and 6).

- Cycles:

(C1) Aggregate investment and household consumption tended to co-move (Figure 7).

(C2) Booms and busts of investment and its credits were driven mainly by SOEs.

III. The theoretical model

In this section we construct a theoretical model of the SOE-led economy to explain the facts about trends and cycles discussed in Section II.2. The model economy is populated by two-period lived workers with overlapping generations. Agents work when young and consume their savings when old. Each young agent is endowed with $\overline{L}$ units of labor.

III.1. **Technology.** There are two production sectors and two corresponding types of firms. For tractability, we assume that all firms are state-owned enterprises and the government is the residual claimant on firms' profits. This assumption is consistent with facts T(3) and C(2). The two production sectors differ in capital intensity as well as in demand for bank loans. The first sector is what we call the heavy sector, composed of capital-intensive firms (K-firms) endowed with a capital-intensive technology. The second sector is what we call the light sector, composed of labor-intensive firms (L-firms).

The technologies of the two types of firms have constant returns to scale:

$$Y_t^k = K_t^k,\ Y_t^l = \left(K_t^l\right)^{\alpha} \left(\chi L_t\right)^{1-\alpha},$$

where Y^j, K^j, and L^j denotes output, capital stock, and labor demand for the type-j firm, $j \in \{K, L\}$. We assume that SOEs in the labor-intensive sector ((e.g., firms in the textile industry) is less productive than the capital-intensive sector ($\chi \leq 1$). This assumption is consistent with the fact that SOEs in the light sector during 1979-1997 were typically small and medium sized.[1] To focus on endogenous growth, exogenous technological growth in both sectors is assumed to be constant (normalized by one).

Final goods are generated from a CES aggregator of the above two intermediate goods:

$$Y_t = \left[\varphi \left(Y_t^k\right)^{\frac{\sigma-1}{\sigma}} + \left(Y_t^l\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The perfect competitiveness in the final goods market implies the following first-order condition

$$\frac{Y_t^k}{Y_t^l} = \varphi^{\sigma} \left(\frac{P_t^l}{P_t^k}\right)^{\sigma}. \tag{1}$$

Normalizing the final goods price to one and using the zero profit condition for final goods, we have

$$\left[\varphi^{\sigma} \left(P_t^k\right)^{1-\sigma} + \left(P_t^l\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1. \tag{2}$$

We calibrate the value σ to be greater than one in Table 1 to be consisten with the empirical finding of Chang, Chen, Waggoner, and Zha (2016) for China. Unlike Chang, Chen, Waggoner, and Zha (2016), however, all our theoretical results hold no matter whether σ is greater or less than one.

III.2. **SOEs in the heavy sector.** For simplicity, we assume that the representative K-firm lives for only one period, an assumption that can be relaxed without affecting our results. At the end of each period, the new born K-firm receives an amount of net worth N_t from the government. The K-firm borrows from their representative financial intermediary at a fixed (gross) interest rate R to finance the gap between its capital investment and the government's net worth. Since both the K-firm and the bank belong to the government,

[1] The less productive light sector was one of the justifications for China's "grasp the large and let go of the small" reforms on SOEs in the later period of the SOE-led economy to reduce excess capacity problems in small and medium-sized SOEs. The theoretical results discussed below, however, do not rely on this assumption.

there is no agency friction between the K-firm and the bank. The problem of the K-firm can be summarized as

$$\Pi_t^k \equiv \max_{K_t^k} P_t^k K_t^k - R\left(K_t^k - N_t\right) + (1-\delta) K_t^k,$$

where P_t^k is the price for the K-firm's output, Π_t^k is the sum of the K-firm's profit and the capital stock after the depreciation, and δ is the depreciation rate. Denote $B_t^k = K_t^k - N_t$ as the investment loan. After the production, the K-firm dies and the government resumes the remaining assets Π_t^k. The optimality condition implies

$$P_t^k = R. \tag{3}$$

III.3. **SOEs in the light sector.** L-firms live for two periods. Before its production takes place, the L-firm must finance its working capital to pay wages ($w_t L_t$) and rent the physical capital from the bank. The rent, $RK_t^l - K_t - \delta K_t$, is subsidized by the government with $N_t - K_t^l - \delta K_t^l$. The remaining portion of the rent, $RK_t - N_t$, and labor costs $w_t L_t$ are financed by the bank's intra-period loan with the interest rate R_t^l.

We model L-firms differently from K-firms to capture the institutional facts in China. During the period of 1978-1997, SOEs in the light sector differed qualitatively from SOEs in the heavy sector. Unlike those in the heavy sector, many SOEs in the light sector were not directly controlled or owned by the central government; rather, they were either controlled by local governments or in the form of collectively owned enterprises in the urban area or township-village enterprises in the countryside around the urban area. These SOEs are nonetheless supported by the central government and modeled as the subsidy $N_t - K_t^l - \delta K_t^l$.

The phase 1978-1997 marked an era in which China underwent a transition from the completely planned economy to the market-oriented economy. During this transition, the government's priority was given to stimulating production of consumer durables in the light sector as a way to generate demands for goods produced by the heavy sector ("the light sector led the heavy sector" was the Chinese slogan during that period). Managers were hired to run SOEs in the light sector. This reform also generated an incentive problem as SOEs were still owned by the government, not by managers. To model this incentive problem, we follow Gertler and Kiyotaki (2010) by assuming that the L-firm may default on its loan payment as the manager may "run away with" a fraction of $1-\theta_t$ of its output Y_t^l where $1 > \theta_t \geq 0$. The incentive compatibility (IC) constraint for the L-firm is

$$P_t^l \left(K_t^l\right)^\alpha (\chi L_t)^{1-\alpha} - R_t^l \left(w_t L_t + RK_t^k - N_t\right) \geq (1-\theta_t) P_t^l \left(K_t^l\right)^\alpha (\chi L_t)^{1-\alpha}. \tag{4}$$

In comparison, SOEs in the heavy sector do not face the borrowing constraint as in the light sector because banks, as part of the government, are willing to make intertemporal (long-term) investment loans without conditions. From the bank's point of view, investment loans can be fully pledged by equipments, land, and real estate in general. Loans to SOEs in the light sector are of short term and used to fund the working capital and the government does not fully guarantee the payments of short-term loans. Consequently, SOEs in the light sector face the borrowing constraint governed by a fraction (θ_t) of its collateral.[2]

The L-firm's problem in the second period can be expressed as

$$\Pi_t^l \equiv \max_{K_t^l, L_t} P_t^l \left(K_t^l\right)^\alpha \left(\chi L_t\right)^{1-\alpha} - R_t^l \left(w_t L_t + R K_t^l\right) + (1-\delta) K_t^l \tag{5}$$

subject to (4). Note that N_t does not enter the L-firm's profit function because the government's subsidy is only for the intra-period. At the end of the period, RK_t^l is returned to the bank and $R^l N_t$ is returned to the government. As wages and capital costs enter the IC constraint symmetrically, first order conditions for the above L-firm's problem give the L-firm's demand for labor as

$$L_t = \frac{(1-\alpha) R K_t^l}{\alpha w_t}, \tag{6}$$

no matter whether constraint (4) is binding or not. Substituting out L_t in the L-firm's production function with (6), we have

$$Y_t^l = K_t^l \left(\frac{(1-\alpha)\chi R}{\alpha w_t}\right)^{1-\alpha}. \tag{7}$$

As we will show later, when there is a labor surplus, the wage rate will be constant. In this case, the production function of the L-firm is linear in K_t^l according to (7)—the AK feature.

Because the IC constraint may be binding, the L-firm's labor demand L_t may be smaller than the labor endowment in the first period (when young). We assume $\overline{L}$ is sufficiently large such that $L_t < \overline{L}$ along the transition path, consistent with the fact that China experienced a labor surplus in the urban area during 1979-1997. Without loss of generality, we also assume that capital depreciates completely in one period ($\delta = 1$).

When the IC constraint is not binding, first-order conditions imply

$$P_t^l = R_t^l \left(\frac{R}{\alpha}\right)^\alpha \left(\frac{w_t}{(1-\alpha)\chi}\right)^{1-\alpha}. \tag{8}$$

When the IC constraint is binding, it must be true that

$$P_t^l \geq R_t^l \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{w_t}{(1-\alpha)\chi}\right)^{1-\alpha}.$$

Proposition 1. When $K_t^l > \frac{\alpha}{R}\frac{N_t}{(1-\theta_t)}$, the IC constraint (4) is binding.

Proof. See Appendix A.1 □

In the remainder of the paper, we consider only the case in which SOEs in the light sector demand large investment such that $K_t^l > \frac{\alpha}{R}\frac{N_t}{(1-\theta_t)}$. The assumption of a large demand of investment in the light sector captures the government's credit policy toward promoting this

[2]After 1997, the "grasp the large and let go of the small" reform on SOEs shifted its focus to the heavy sector that received preferential credits from the government. As the reform moved to a more market-oriented economy and firms (SOEs and non-SOEs alike) in the heavy sector were run by managers, an incentive problem similar to (4) arose but this time in the heavy sector. This incentive problem is modeled explicitly in Chang, Chen, Waggoner, and Zha (2016), who study the post-1997 economy.

sector during the SOE-led economy. In our model, this credit policy is composed of two components: the trend (long-term) part and the cycle part (policy shocks). The trend part of the credit policy is reflected in the form of N_t assisting SOEs in the light sector. The time varying variable θ_t captures the cyclical part of the government's credit policy. As shown in Section??????, a positive shock to θ_t raises the credit volume relative to aggregate output and helps increase both consumption and investment, a result consistent with the empirical observations displayed in Figures??????.

With constraint (4) binding, we substitute (6) into (4) and obtain the demand function for K_t^l as

$$K_t^l = \frac{\alpha}{R} \frac{R_t^l N_t}{R_t^l - \theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha}}, \tag{9}$$

and the loan demand for the working capital as

$$B_t^l = \frac{\theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha}}{R_t^l - \theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha}} N_t. \tag{10}$$

III.4. **Workers' problem.** We assume that workers cannot lend directly to the firms, but can deposit their savings in the representative bank and earn a fixed interest rate R. The representative worker's consumption-saving problem is

$$\max_{c_{1t}^w, c_{2t+1}^w} \frac{\left(c_{1t}^w - \underline{w}\left(L_t - \overline{L}\right)\right)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \beta \frac{\left(c_{2t+1}^w\right)^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}$$

subject to

$$\begin{aligned} c_{1t}^w + s_{t+1}^w &= w_t L_t, \\ c_{2t+1}^w &= s_{t+1}^w R, \\ L_t &\leq \overline{L}, \end{aligned}$$

where $\underline{w}(\overline{L} - L_t)$ represents consumption from the home production, $\underline{w}$ is the reservation wage rate, w_t is the market wage rate, and c_{1t}^w, c_{2t+1}^w, and s_t^w denote the worker's consumption when young, the worker's consumption when old, and the worker's savings. The first order condition for labor supply, together with the constraint on labor endowment, implies the kinked labor supply curve as

$$w_t = \underline{w} \text{ for } L_t < \overline{L},$$

$$w_t \geq \underline{w} \text{ for } L_t = \overline{L}.$$

The worker's labor supply is perfectly elastic at $\underline{w}$ until it is binding at $\overline{L}$. In 1978-1997, there was abundant labor supply in the urban area such that $L_t < \overline{L}$. Real wages in 1978-1997 grew at a rate that was only half of the wage growth rate in 1998-2016, which can be accounted for by exogenous TFP growth.[3] Our model focuses on endogenous TFP growth engineered within the light sector and the impact of such growth on the aggregate economy.

III.5. **The bank's problem.** Every period the bank receives deposits, D_t, from young workers. These savings are channeled to both intertemporal lending to K-firms for long-term investment and intratemporal lending to L-firms for funding the working capital. The bank's interest rate for investment loans equals R, but the interest rate for short-term loans to the working capital is R_t^l. The remaining deposits, $D_t - B_t^k - K_t^l$, are invested in foreign bonds and earn interest at a rate of R.[4] The bank's problem can be described as

$$\Pi_t^b \equiv \max_{B_t^l} R_t^l B_t^l + R(B_t^k + K_t^l) + R(D_t - B_t^k - K_t^l) - RD_t - B_t^l,$$

where B_t^k is an intertemporal loan to investment for the K-firm and B_t^l is an intratemporal loan to the working capital for the L-firm. The optimality condition gives

$$R_t^l = 1. \tag{11}$$

In equilibrium, $D_t = B_t^w + B_t^k + K_t^l$, where B_t^w is part of workers' deposits invested by the bank in foreign bonds, which equals $s_t^w - B_t^k - K_t^l$.

III.6. **The government's problem.** The government is infinitely-lived. The government provide guarantees to SOEs in both light and heavy sectors with its net worth N_t. The government receives SOE profits from both sectors and purchases foreign assets with the international interest rate R as government savings (or issues government bonds if the savings are negative). At the end of each period, the government decides on how much of the net worth to be advanced to K-firms in the next period. For simplicity, we assume that the government's net worth N_{t+1} in the next period is a fraction ξ of the current period capital stock of K-firms, i.e.,

$$N_{t+1} = \xi K_t^k. \tag{12}$$

The government's budget constraint is

$$B_{t+1}^G + N_{t+1} = \Pi_t^k + \Pi_t^l + \Pi_t^b + RB_t^G, \tag{13}$$

where B_t^G represents government assets invested in foreign bonds at the beginning of the period with the fixed interest rate R.

[3]Even for the period after 1997, Song, Storesletten, and Zilibotti (2011) (SSZ hereafter) assume a constant rate of wage growth to emphasize the role of endogenous TFP growth.

[4]B_t^l is an intratemporal loan, which is paid in full at the end of the period.

III.7. **Equilibrium conditions.** Below we list all the equilibrium conditions:

$$L_t = \frac{(1-\alpha) R K_t^l}{\alpha w_t}, \tag{14}$$

$$\Pi_t^l = P_t^l \left(K_t^l\right)^{\alpha} (\chi L_t)^{1-\alpha} - R_t^l \left(w_t L_t + R K_t^l\right) + (1-\delta) K_t^l, \tag{15}$$

$$K_t^l = \frac{\alpha}{R} \frac{R_t^l N_t}{R_t^l - \theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha}}, \tag{16}$$

$$R_t^l = 1, \tag{17}$$

$$\Pi_t^k = P_t^k K_t^k - R\left(K_t^k - N_t\right) + (1-\delta) K_t^k, \tag{18}$$

$$\Pi_t^B = \left(R_t^l - 1\right) B_t^l, \tag{19}$$

$$B_t^k = K_t^k - N_t, \tag{20}$$

$$B_t^l = w_t L_t + R K_t^l - N_t, \tag{21}$$

$$N_{t+1} = \xi K_t^k, \tag{22}$$

$$Y_t = \left[\varphi \left(Y_t^k\right)^{\frac{\sigma-1}{\sigma}} + \left(Y_t^l\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{23}$$

$$Y_t^k = K_t^k, \tag{24}$$

$$Y_t^l = Y_t^l = \left(K_t^l\right)^{\alpha} (\chi L_t)^{1-\alpha}, , \tag{25}$$

$$1 = \left[\varphi^{\sigma} \left(P_t^k\right)^{1-\sigma} + \left(P_t^l\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{26}$$

$$P_t^k = R, \tag{27}$$

$$B_{t+1}^G = \Pi_t^k + \Pi_t^l + \Pi_t^b + R B_t^G - N_{t+1} + \left(R_t^l - 1\right) N_t, \tag{28}$$

$$P_t^l = \frac{P_t^k}{\varphi} \left(\frac{Y_t^k}{Y_t^l}\right)^{\frac{1}{\sigma}}, \tag{29}$$

$$s_{t+1}^w = w_t / \left(1 + \beta^{-\gamma} R^{1-\gamma}\right), \tag{30}$$

$$B_t^w = s_t^w - B_t^k - K_t^l, \tag{31}$$

$$c_{1t}^w = w_t - s_{t+1}^w, \text{ and } c_{2t}^w = s_t^w R. \tag{32}$$

Combining the budget constraints of households, firms, and the government, we obtain the following resource constraint

$$C_t + I_t + S_t^{\mathrm{CA}} = Y_t^{\mathrm{GDP}} = Y_t + (R-1)\left(B_t^w + B_t^G\right), \tag{33}$$

where

$$\begin{aligned} C_t &= c_{1t}^w + c_{2t}^w, \\ I_t &= K_{t+1} - (1-\delta) K_t, \\ K_t &= K_t^k + K_t^l, \\ S_t^{\mathrm{CA}} &= B_{t+1}^w + B_{t+1}^G - \left(B_t^w + B_t^G\right). \end{aligned}$$

IV. Characterizing the transitional path

In this section, we study the dynamic patterns of various macroeconomic variables along the transitional path in which $L_t \leq \overline{L}$ and the wage rate is constant ($w_t = \underline{w}$). To obtain the closed-form solution and without loss of generality, we assume $\delta = 1$.

IV.1. **Dynamics in and between the two sectors.** The revenue ratio between the two sectors is

$$\frac{P_t^k Y_t^k}{P_t^l Y_t^l} = \left(\frac{P_t^k}{P_t^l}\right)^{1-\sigma} \varphi^{\sigma}.$$

One can see from equations (3) and (26) that the revenue ratio between the two sectors is constant. There is no resource reallocation between the two sectors along the transitional path.

The labor income share is also constant. The labor income in our model is the sum of the wage income and the $1-\alpha$ fraction of the L-firm's profit Π_t^l:

$$\frac{w_t L_t + (1-\alpha)\,\Pi_t^l}{Y_t} = (1-\alpha)\,\frac{1}{1 + P_t^k Y_t^k / \left(P_t^l Y_t^l\right)}. \tag{34}$$

Since the revenue ratio $\frac{P_t^k Y_t^k}{P_t^l Y_t^l}$ is constant, the labor income share is constant.

The ratio of capitals (or investments) between the two sectors is also constant. Substituting (1) and (6) into the production functions in both sectors, we have

$$\frac{K_t^k}{K_t^l} = \varphi^{\sigma} \left(\frac{\chi\,(1-\alpha)\,R}{\alpha w_t}\right)^{1-\alpha} \left(\frac{P_t^l}{P_t^k}\right)^{\sigma}, \tag{35}$$

which is constant when $w_t = \underline{w}$ for $L_t \leq \overline{L}$.

The ratio of investment loan to working capital loan can be computed as follows

$$\frac{B_t^k}{B_t^l} = \frac{K_t^k - N_t}{\frac{R}{\alpha} K_t^l - N_t}, \tag{36}$$

where we have used equation (6). For similar logic as above, we can see that the ratio of bank loan of the capital intensive sector to that of labor intensive sector decreases over time.

Transitional dynamics in each of the two sectors can be characterized by the AK feature.

Proposition 2. Capital and labor K_t^l, K_t^k and L_t have the same balanced growth rate:

$$\frac{K_{t+1}^l}{K_t^l} = \frac{K_{t+1}^k}{K_t^k} = \frac{L_{t+1}}{L_t} \equiv g_A,$$

where

$$g_A = \frac{\xi \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha} \varphi \left(\frac{P^l}{P^k}\right)^{\sigma} R^l}{R^l - \theta P^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^{\alpha}}.$$

Proof. See Appendix A.2. □

From Proposition 2 one can derive the aggregate investment rate and the ratio between bank loans to the two sectors. The aggregate investment rate is a weighted average of investment rates across the two sectors:

$$\frac{I_t}{Y_t} = \frac{I_t^k}{P_t^k Y_t^k}\frac{P_t^k Y_t^k}{Y_t} + \frac{I_t^l}{P_t^l Y_t^l}\frac{P_t^l Y_t^l}{Y_t}, \tag{37}$$

where the weight equals the revenue share of each sector. Because the revenue share in each sector is constant, the aggregate investment rate is constant according to equation (37) as long as the investment rate in each sector (i.e., $\frac{I_t^k}{P_t^k Y_t^k}$ or $\frac{I_t^l}{P_t^l Y_t^l}$)) is constant. We have the following proposition.

Proposition 3. The investment rate in each sector and the ratio between bank loans to the two sectors are constant along the transitional path.

Proof. See Appendix A.3. □

During the transition in which $w_t = \underline{w}$, it follows from equation (7) that the production technology in the light sector is of AK. Since the production technology in the heavy sector is also of AK, the investment rate in each sector must be constant. This result, together with the constant revenue share of the final output in each sector, implies that the aggregate investment rate is also constant. The ratio of bank loans to both sectors is constant because the capital stock in each sector and therefore bank loans to each sector are proportionate to the government's net worth. This result is robust to a case in which the depreciate rate is less than one. In this general case, investment in each sector remains proportionate to the capital stock in its own sector:

$$I_t^j = \left(g_A - (1-\delta)\right) K_t^j, \quad \text{for } j \in \{k, l\}.$$

IV.2. **Endogenous TFP growth.** The growth accounting discussed in Section ?????? reveals that that TFP growth is the main driver of the SOE-led economy. Our theoretical model's prediction is consistent with this empirical finding. With the Cobb-Douglas production function, our model's aggregate TFP is calculated in the same way as in our data:

$$TFP_t = \frac{Y_t}{K_t^\eta \overline{L}^{1-\eta}}, \tag{38}$$

where $1-\eta$ is the aggregate labor income share as measured in the data. Thus, it must be $\eta > \alpha$ because $1-\alpha$ is the labor income share for the light (labor-intensive) sector. Note that instead of $L_t, \overline{L}$ enters the denominator of the definition of TFP. In China, there is no data on hours worked and the employment is used to compute aggregate TFP in the data. When we calculate the aggregate TFP, the employment in the data corresponds to the total labor endowment in our model rather than actual hours worked.

Proposition 4. The aggregate TFP can be expressed as

$$TFP_t = A\left(\frac{L_t}{\overline{L}}\right)^{1-\eta},$$

where A is a constant. As a result, the aggregate TFP grows at a constant rate of $g_A^{1-\eta}$.

Proof. See Appendix A.4. □

SSZ have a two-sector model describing China's economy since late 1990s. Our model differs from theirs in two critical aspects. First, in SSZ the aggregate investment rate declines despite the AK feature of the production function for a particular transition emphasized by SSZ. In that transition, labor is reallocated from the SOE sector with a higher investment rate to the POE sector with a lower investment rate (POEs) while POEs accumulates capital. We assume that all firms are SOEs as a reasonable approximation to the 1978-1997 period, and there is no resource reallocation between the heavy and light sectors. Instead, the labor surplus, a key AK feature during the SOE-led economy, is absorbed by an increase of labor demands by firms in the light (labor-intensive) sector.

Second, the source of TFP growth is different from SSZ. Both models focus on endogenous TFP growth, but its source differs. In SSZ, the labor allocation is efficient and the aggregate TFP growth stems from reallocating capital from the SOE sector to the POE sector. In our model, by contrast, the aggregate TFP growth originates from an improvement of allocative efficiency from the idle household labor to the market production. This mechanism is consistent with the institutional fact that an emergence of township-village and collectively owned enterprises during the early period of China's opening-up reforms attracted the idle labor from households into the market production.

IV.3. **Intuition.** In this section, we provide economic intuition for some key results in the preceding section. For Proposition 2, the rate of returns to capital for L-firms is constant because of the labor surplus during the transition. The linear law of motion for the government's net worth and the constant price of goods produced by K-firms imply that the growth rate of L-firms' capital is constant. Because capital in K-firms is proportional to capital in L-firms, the growth rate of capital in K-firms must be as well. The same intuition applies to the growth rate of demand for L-firms' labor.

The intuition for the growth of endogenous TFP (Proposition 4) is straightforward. As demands for labor in L-firms rise with an increase of the government's net worth, more labor hours per worker are reallocated from home production to market production. This reallocation raises the aggregate output per worker (labor productivity) and therefore the aggregate TFP. The TFP in the light sector is

$$TFP_t^l = \frac{P_t^l Y_t^l}{\left(K_t^l\right)^\alpha \overline{L}^{1-\alpha}}.$$

Similar to the proof of Proposition 4, one can show that

$$\frac{TFP_{t+1}^l}{TFP_t^l} = g_A^{1-\alpha}.$$

Since $1-\alpha > 1-\eta$, we have

$$\frac{TFP^l_{t+1}}{TFP^l_t} > \frac{TFP_{t+1}}{TFP_t}.$$

Thus, the source of the aggregate TFP growth is the growth of endogenous TFP in the light (labor-intensive) sector, consistent with the empirical findings in the existing literature.[5]

When the government expands its credit policy, the expansionary policy has a direct impact on the light sector as L_t increases immediately. The Cobb-Douglas production in the light sector implies that L-firms increase K_t accordingly. Consequently, output Y^l_t in the light sector rises. According to the equilibrium condition that

$$Y^k_t = \phi^\sigma \left(\frac{P^l_t}{P^k_t}\right)^\sigma Y^l_t,$$

where $P^k_t = R$ and $P^l_t = 1$, output Y^k_t in the heavy sector sector increases as well. It can be seen that the light sector plays a leading role in an increase of aggregate output following the government's expansionary credit policy, consistent with what the Chinese literature calls "the light sector leading the heavy sector."[6]

V. Quantitative Results

In this section we show by simulation that our model results are consistent with the facts of trends and cycles in the SOE-led economy. Table 1 reports the parameter configuration used for simulation. Under this configuration, the collateral constraint for L-firms is bind at the steady state.

V.1. **Trends.** Assuming that the initial net worth of the government is below the steady state value, we simulate a path of the transition to the steady state. In the simulation, we keep θ constant along the transition. In the next section we allow for the fluctuation of θ_t to obtain the simulated cyclical patterns.

Figure 10 shows the transitional paths of labor inputs, wages, the aggregate investment rate, the capital and loan ratios between the heavy and light sectors, and the aggregate TFP. The transition lasts until all the labor endowment is employed by L-firms. During the transition, labor demands by L-firms increase monotonically (Panel A) while the wage rate is constant (Panel B). Because of the AK feature, the aggregate investment rate and the capital ratio between the heavy and light sectors are constant (Panels C and D). Since bank loans in both sectors are proportional to the government's net worth, the loan ratio between the two sector is also constant (Panel E). As we show in Section IV.1, the revenue ratio between the two sectors is constant and therefore the labor share is constant as well. As more labor is hired by L-firms, TFP grows along the transition (Panel F).

[5]See the Brookings opinion article "Future Path of China's SOE Reforms" (in Chinese) available at http://brook.gs/2bPJHBB.

[6]See Lin (2013) and the Brookings opinion article "Future Path of China's SOE Reforms" (in Chinese) available at http://brook.gs/2bPJHBB.

The simulated results reported in Figure 10 accord with the observations discussed in Sections **??**????. They are in sharp contrast to the stylized facts documented by Chang, Chen, Waggoner, and Zha (2016) for the post-1997 period. Our theoretical model, together with the model of Chang, Chen, Waggoner, and Zha (2016), provides a coherent explanation of how China went through the drastic regime change and how such a change affected the trend and cycle of China's macroeconomy.

V.2. **Cycles.** We now explore the cyclical behavior of the model by calculating the impulse responses to a shock to the credit quota, that is, an unexpected increase in θ_t. Since the SOE-led economy is on the transitional path, the cyclical fluctuation of each variable is calculated as a percentage deviation from the transitional path in the absence of any shocks, rather than a deviation from the steady state.

We assume that a credit shock follows an AR(1) process with the persistence parameter 0.96^{30} and Panel A of Figure 11 displays the path of θ_t in response to this shock. Other panels in the figure display various impulses responses to the credit shock. It can be seen from Panels B and C that both aggregate consumption and aggregate investment increase in response to a credit expansion, comoving positively. An increase of θ_t relaxes the credit constraint on L-firms, raises working capital loans in the light sector, and consequently aggregate bank loans (Panel D). A relaxation of the credit constraint on SOEs in the light sector boosts labor demands, pushes up the household's labor income (Panel E), and increases output in both light and heavy sectors. As a result, aggregate output rises in response to a positive shock to credit (Panel F). All these results are consistent with the empirical findings discussed in Sections **??**?????.

Figure 11 compares the trend along the transitional path and the fluctuation around the trend in response to a credit shock for the two important variables: the ratio of aggregate bank loans to aggregate output and the ratio of aggregate investment to aggregate output (the investment rate). Although both ratios are constant along the transitional path, they increase in response to a temporary increase of credits. The constant ratios are consistent with the trend pattern in the data (i.e., the stationary investment rate), while the responses accord well with the cyclical increase of the investment rate observed during the credit boom (Figure **??**????).

Table 1. Parameter values

Parameter	Definition	Value
α	Capital income share in the light sector	0.40
η	Aggregate capital income share (derived)	0.6615
β	Subjective discount factor	$(0.96)^{30}$
ξ	Government's net worth accumulation	0.56
θ	Leverage ratio for the heavy sector	0.40
δ	Capital depreciation rate	1.0
χ	Relative labor-augmented TFP level in the light sector	0.8
σ	Elasticity of substitution between the heavy and light sectors	2.0
R	Interest rate for investment loans to the heavy sector	1.05
φ	Share of heavy-sector output in total (final) output	0.67
γ	Intertemporal elasticity of substitution	1.0

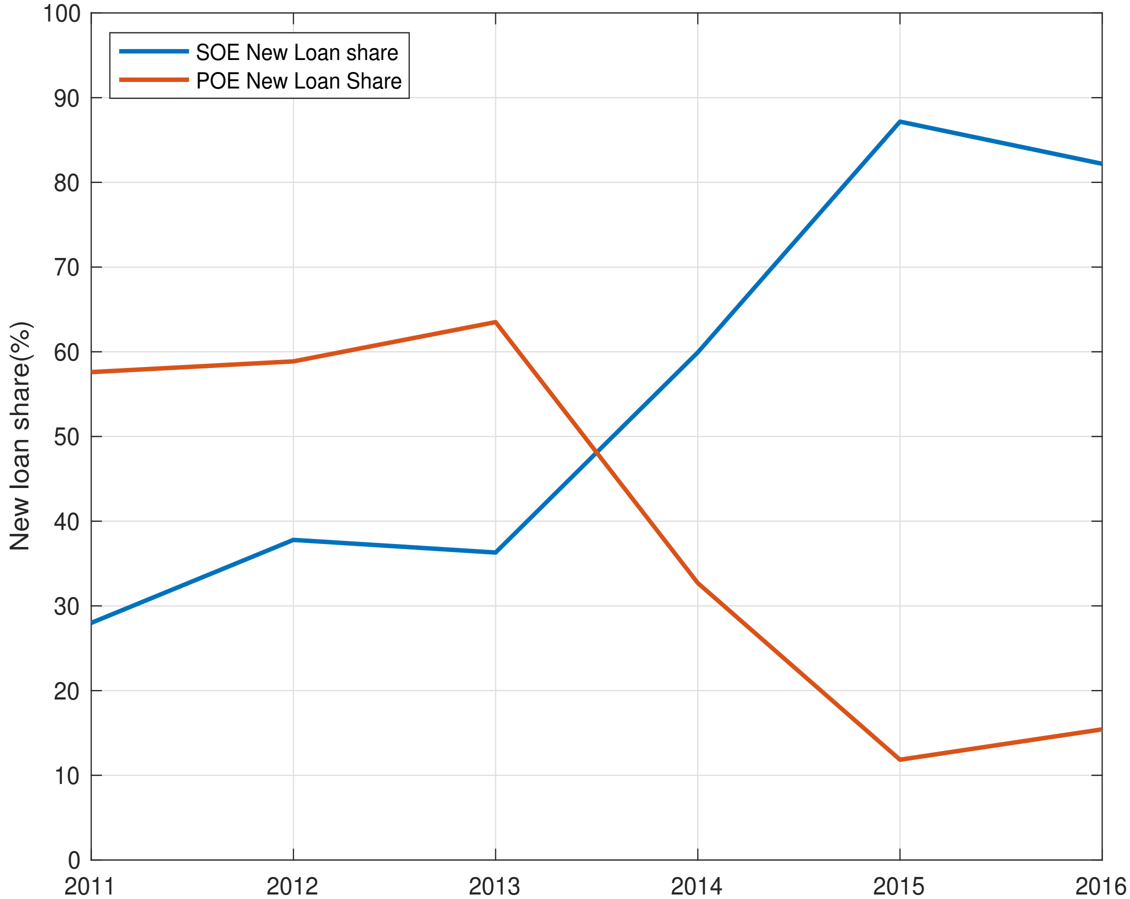

FIGURE 1. The share of newly issued bank loans to SOEs and POEs in total newly issued bank loans.

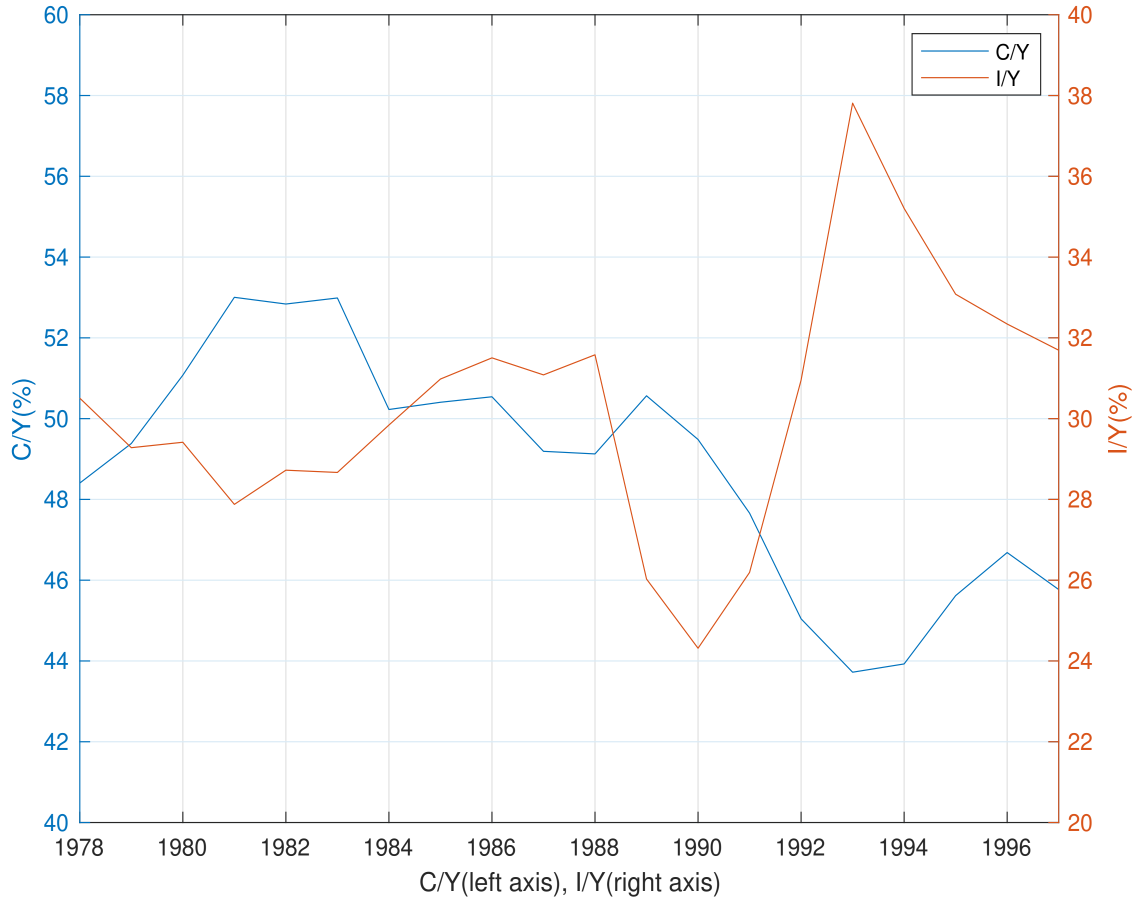

Figure 2. Ratios of investment and consumption to GDP.

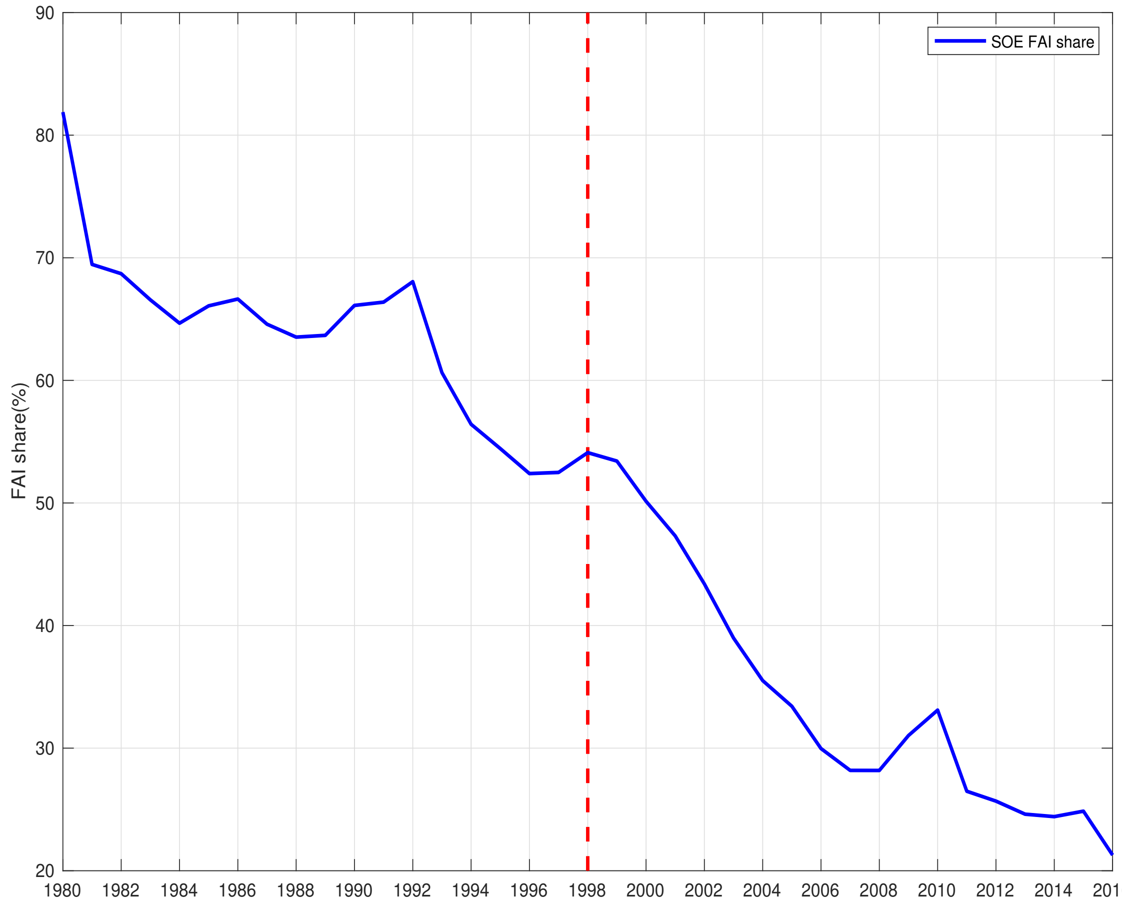

FIGURE 3. Share of SOEs in FAI.

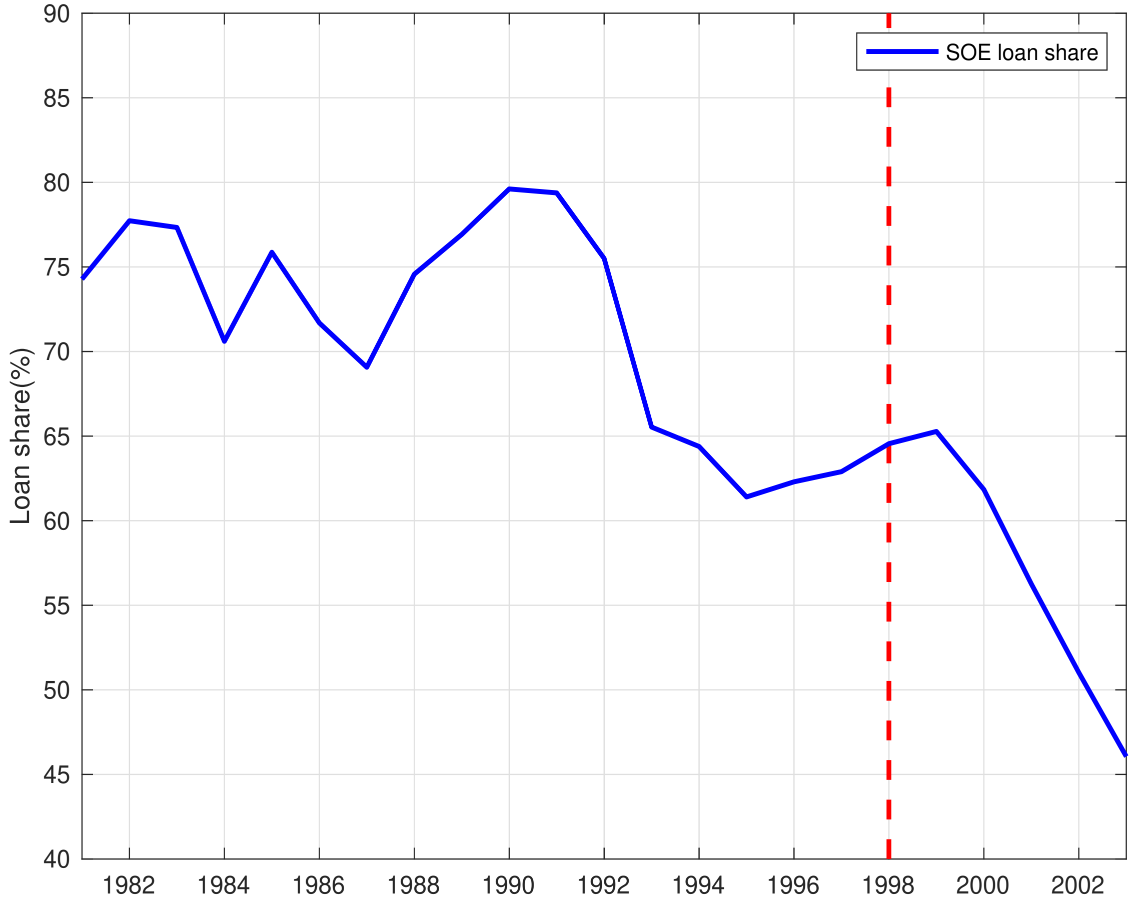

FIGURE 4. Share of SOEs in bank loans to investment.

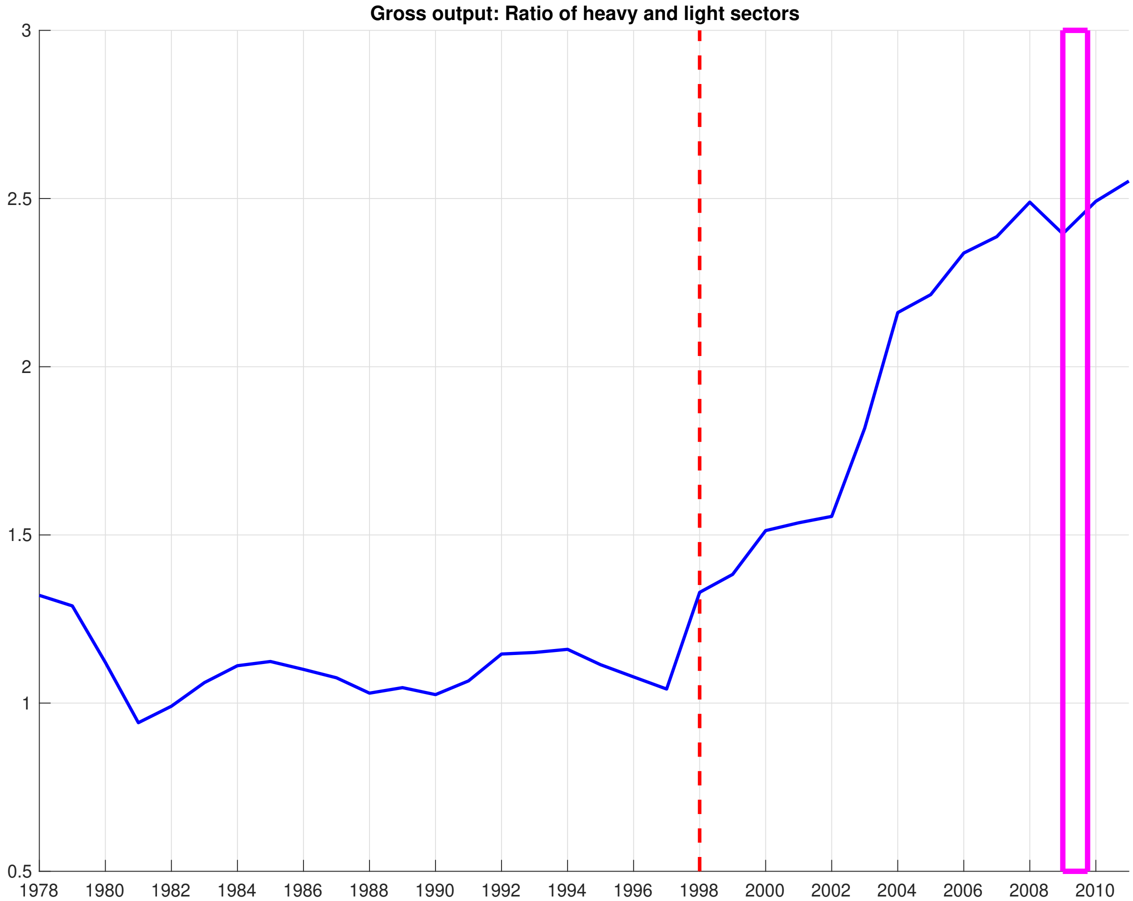

Figure 5. Ratio of gross output in the heavy sector to that in the light sector.

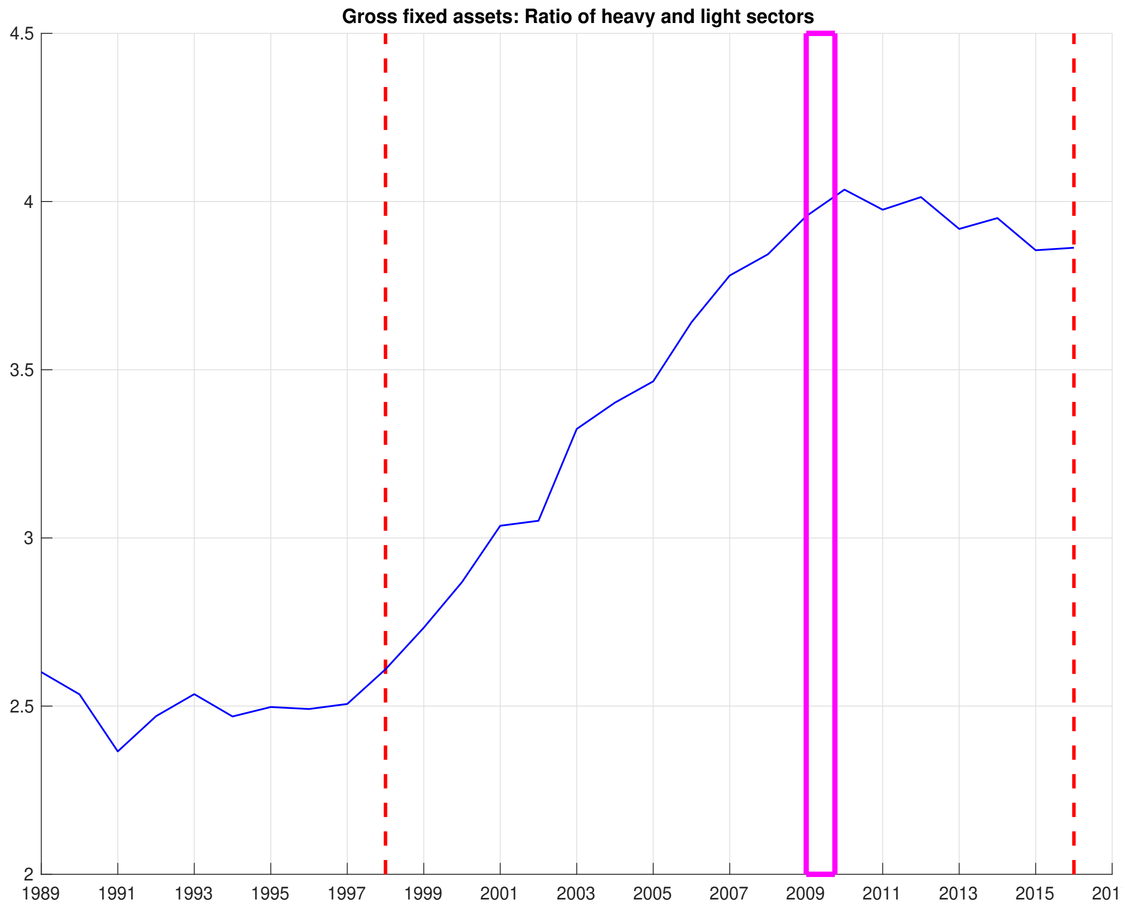

FIGURE 6. Ratio of gross fixed assets in the heavy sector to that in the light sector.

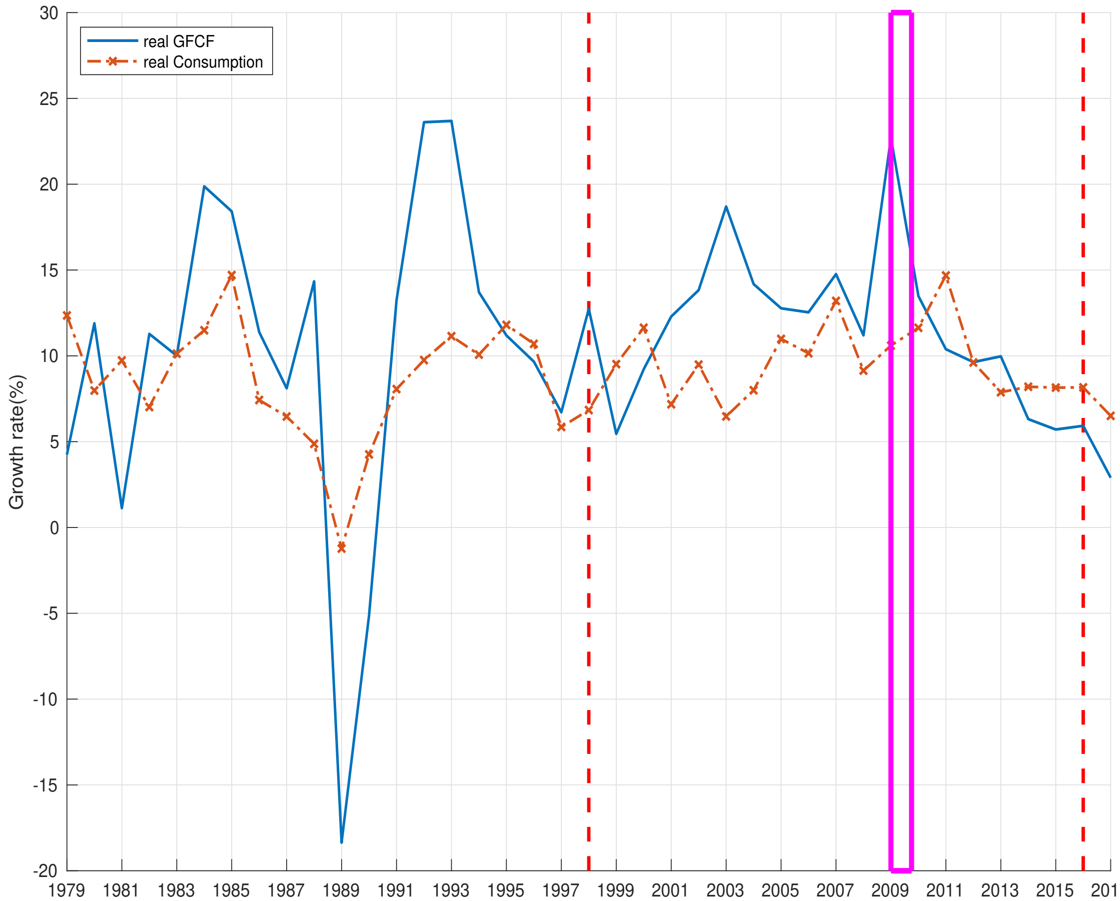

Figure 7. Year-over-year growth of aggregate investment and household consumption.

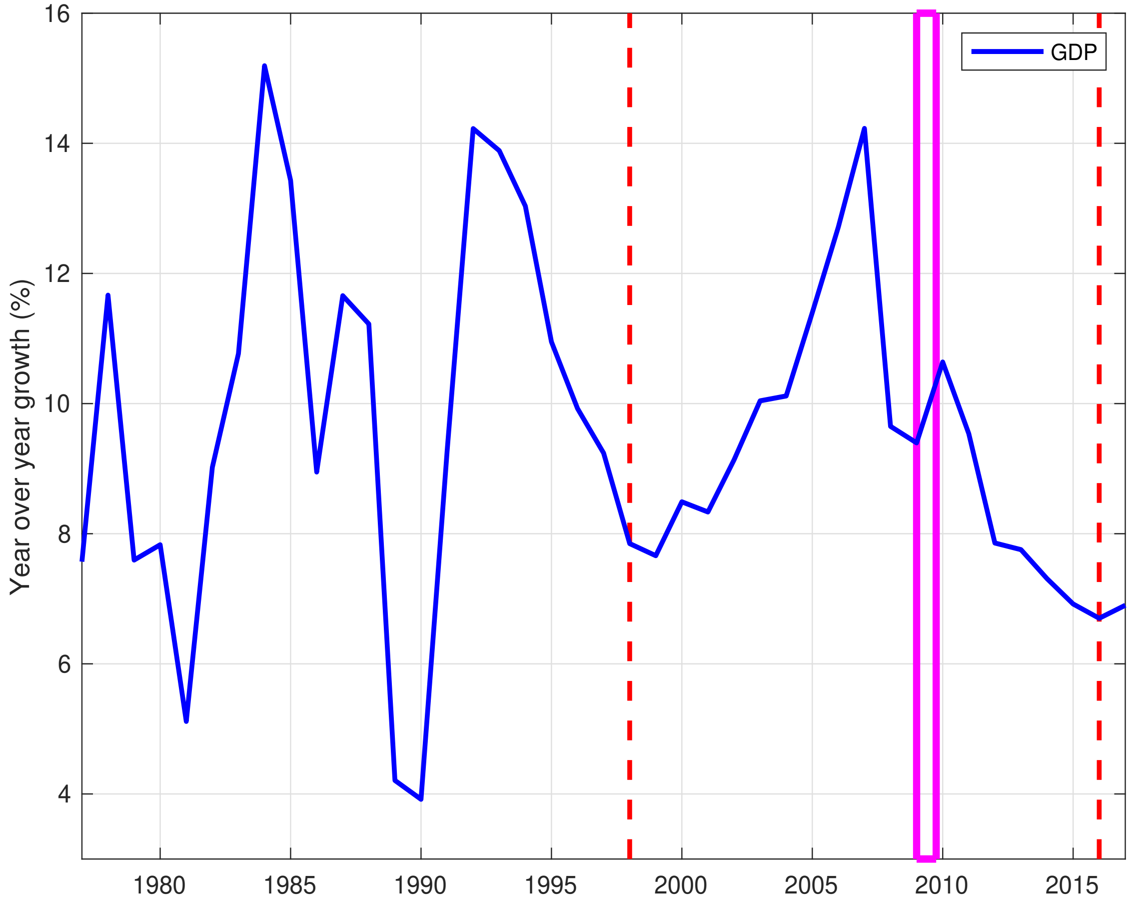

FIGURE 8. GDP growth (annual data).

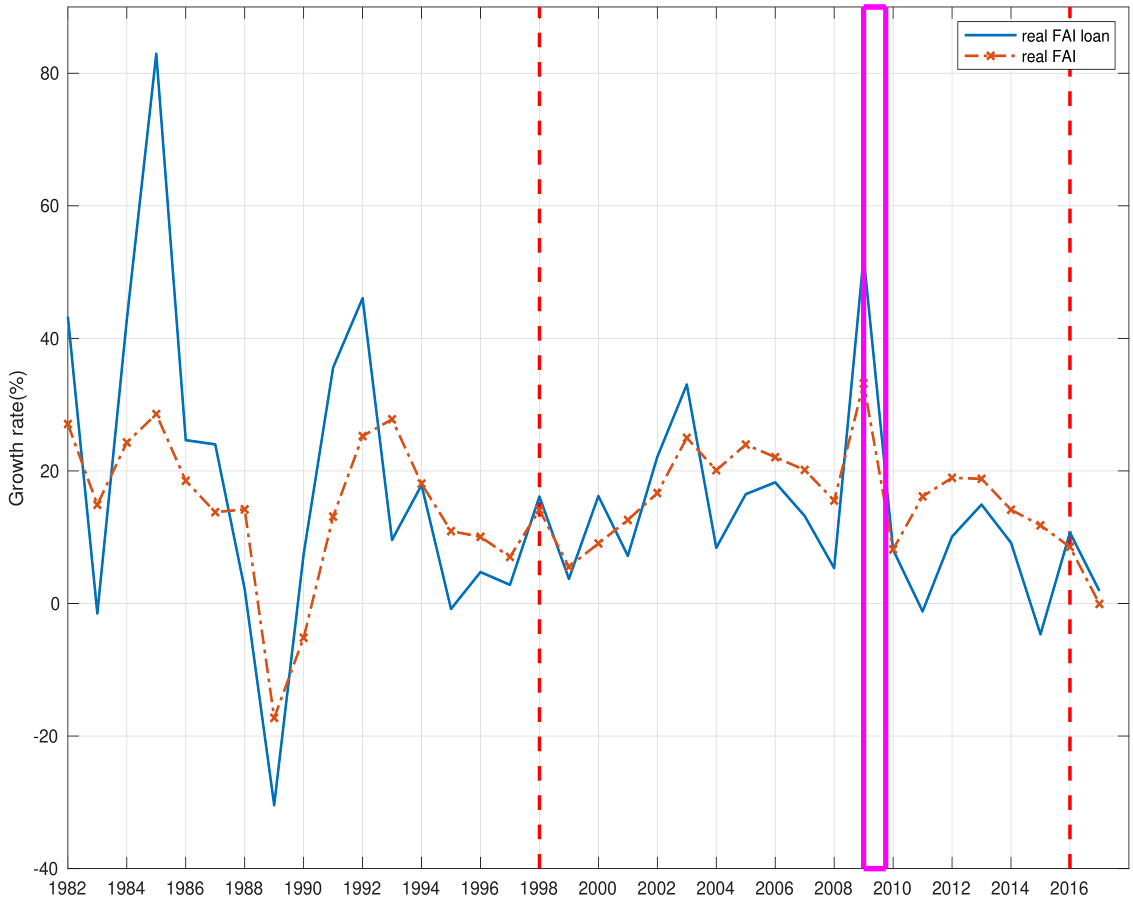

Figure 9. Year-over-year growth rates of FAI and bank loans to FAI.

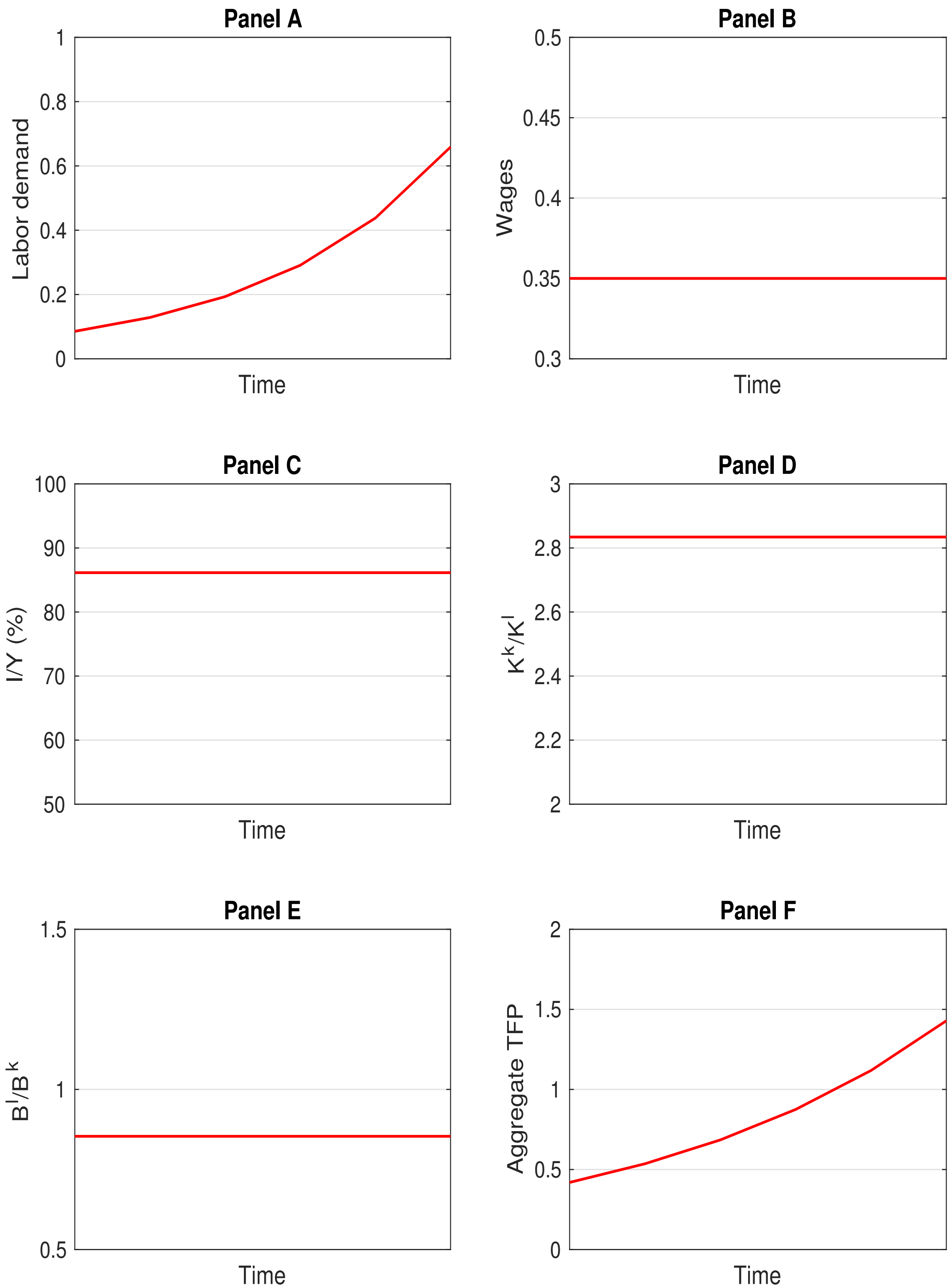

FIGURE 10. The trend pattern for the theoretical model. The symbol I denotes aggregate investment, Y aggregate output, K^k the capital stock in the heavy sector, K^l the capital stock in the light sector, B^k long-term bank loans to investment in capital, and B^l short-term banks loans to funding the working capital.

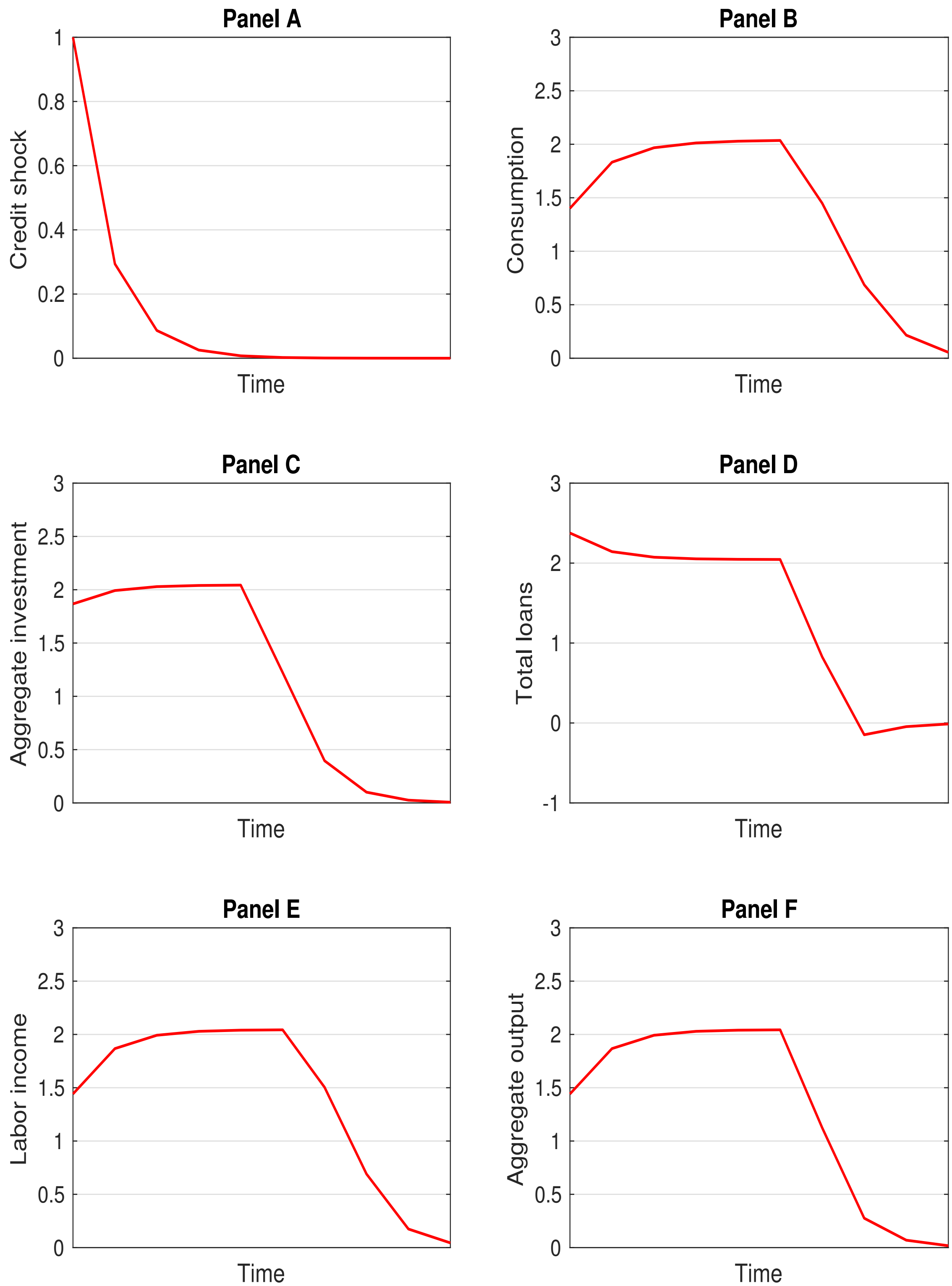

FIGURE 11. Impulse responses (%) to an expansionary credit shock for the theoretical model.

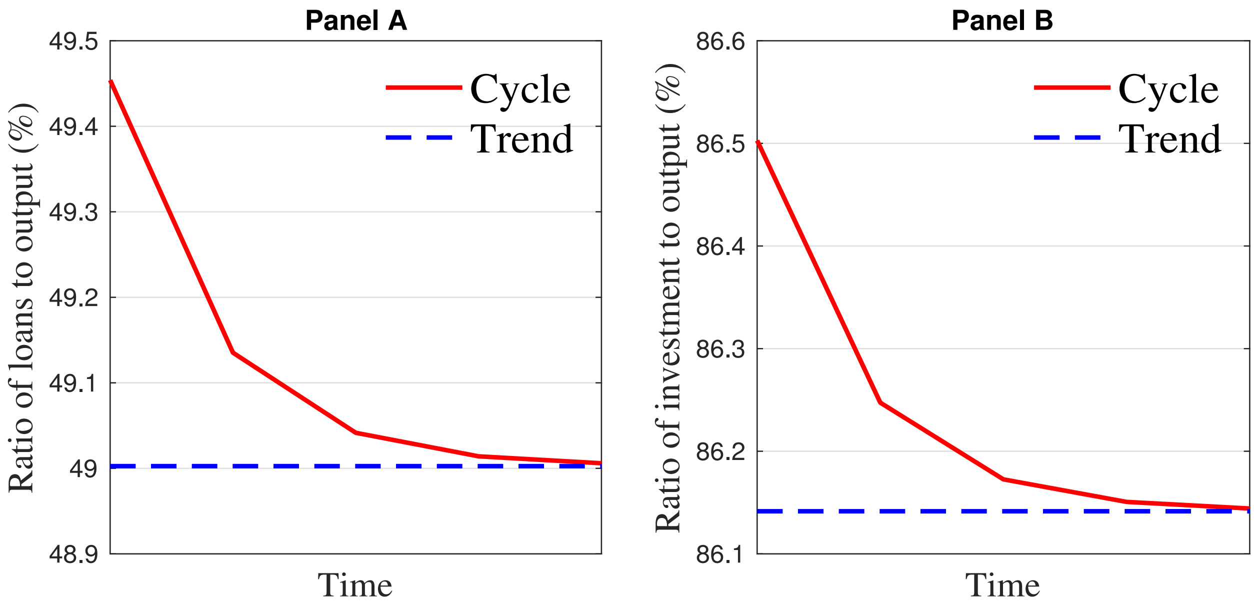

Figure 12. Trends versus impulse responses to an expansionary credit shock for the theoretical model. The magnitude of the responses is small because we use only a 1% increase in the credit shock.

In the appendices below, all labels for equations, tables, definitions, and propositions begin with S, standing for *supplement* to the main text.

Appendix A. Proofs of Propositions

A.1. **Proof of Proposition 1.** We first show that when $N_t = 0$, constraint (4) is binding. By contradiction, suppose the IC constraint is nonbinding. This implies that the following inequality holds:

$$\theta_t P_t^l \left(K_t^l\right)^\alpha \left(\chi L_t\right)^{1-\alpha} \geq R_t^l \left(w_t L_t + R K_t^k\right).$$

If (4) is not binding, however, $P_t^l \left(K_t^l\right)^\alpha \left(\chi L_t\right)^{1-\alpha} = R_t^l \left(w_t L_t + R K_t^k\right)$. With $\theta_t < 1$, the above inequality does not hold unless $\theta_t = 1$. A contradiction.

Now we consider the case in which $N_t > 0$ and the IC constraint is nonbinding. Substituting (6) and (8) into (4), we have

$$K_t^l \leq \frac{\alpha}{R} \frac{N_t}{(1-\theta_t)},$$

and

$$P_t^l = R_t^l \left(\frac{R_t}{\alpha}\right)^\alpha \left(\frac{w_t}{(1-\alpha)\chi}\right)^{1-\alpha}.$$

We again establish a contradiction.

A.2. **Proof of Proposition 2.** Combining equations (12) and (35) leads to

$$N_{t+1} = \xi K_t^l \left(\frac{\chi (1-\alpha) R}{\alpha \underline{w}}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma.$$

Substituting the above equation into (16) and forwarding the resultant equation by one period, we have

$$\begin{aligned} K_{t+1}^l &= \frac{\alpha}{R} \frac{R^l \xi \left(\frac{\chi(1-\alpha)R}{\alpha \underline{w}}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma}{R^l - \theta P^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^\alpha} K_t^l \\ &= g_A K_t^l, \end{aligned}$$

where

$$g_A \equiv \frac{\alpha}{R} \frac{R^l \xi \left(\frac{\chi(1-\alpha)R}{\alpha \underline{w}}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma}{R^l - \theta P^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha} \left(\frac{\alpha}{R}\right)^\alpha}.$$

For K_t^k, it follows from (35) that

$$\frac{K_{t+1}^k}{K_t^k} = \frac{K_{t+1}^l}{K_t^l},$$

From equation (6) one can see that

$$\frac{L_{t+1}}{L_t} = \frac{K_{t+1}^k}{K_t^k}.$$

A.3. **Proof of Proposition 3.** For the light sector, the investment rate is

$$\begin{aligned} \frac{I_t^l}{P_t^l Y_t^l} &= \frac{K_{t+1}^l}{P_t^l Y_t^l} \\ &= \frac{g_A K_t^l}{P_t^l K_t^l \left(\frac{(1-\alpha)\chi R}{\underline{w}\alpha}\right)^{1-\alpha}} \\ &= \frac{g_A}{P^l \left(\frac{(1-\alpha)\chi R}{\underline{w}\alpha}\right)^{1-\alpha}}. \end{aligned}$$

where the second equality comes from Proposition 2 and equation (7).

For the heavy sector, the investment rate is

$$\begin{aligned} \frac{I_t^k}{P_t^k Y_t^k} &= \frac{K_{t+1}^k}{P_t^k Y_t^k}, \\ &= \frac{K_{t+1}^l \left(\frac{\chi(1-\alpha)R}{\alpha w_{t+1}}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P_{t+1}^l}{P_{t+1}^k}\right)^\sigma}{P_t^k Y_t^l \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma}, \\ &= \frac{K_{t+1}^l}{P_t^k K_t^l}, \\ &= \frac{g_A}{P^k}, \end{aligned}$$

where the second equality comes from equations (35) and (1) and the third equality follows from (7). With the constant revenue share in each sector, therefore, the aggregate investment rate is constant.

For bank loans, we substitue (16) and (35) into (36), which yields

$$\begin{aligned} \frac{B_t^k}{B_t^l} &= \frac{\left(\frac{\alpha}{R}\frac{R_t^l \left(\frac{\chi(1-\alpha)R}{\alpha w_t}\right)^{1-\alpha}\varphi^\sigma\left(\frac{P_t^l}{P_t^k}\right)^\sigma}{R_t^l - \theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha} - 1\right) N_t}{\frac{\theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha}{R_t^l - \theta_t P_t^l \left(\frac{(1-\alpha)\chi}{w_t}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha} N_t} \\ &= \frac{R^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha \varphi^\sigma \left(\frac{P^l}{P^k}\right)^\sigma - \left(R^l - \theta P^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha\right)}{\theta P^l \left(\frac{(1-\alpha)\chi}{\underline{w}}\right)^{1-\alpha}\left(\frac{\alpha}{R}\right)^\alpha}. \end{aligned}$$

This ratio is therefore constant during the transition.

A.4. **Proof of Proposition 4.** We rewrite equation (38) as

$$\begin{aligned} TFP_t &= \frac{P_t^k Y_t^k + P_t^l Y_t^l}{\left(K_t^k + K_t^l\right)^\alpha \overline{L}^{1-\alpha}} \\ &= \frac{\left[P_t^k \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma + P_t^l\right] Y_t^l}{\left[K_t^l \left(\left(\frac{\chi(1-\alpha)R}{\alpha w_t}\right)^{1-\alpha} \varphi \left(\frac{P_t^l}{P_t^k}\right)^\sigma + 1\right)\right]^\eta \overline{L}^{1-\eta}} \\ &= \frac{\left[P_t^k \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma + P_t^l\right] \left(\frac{w_t \alpha}{(1-\alpha)\chi R}\right)^\alpha L_t}{\left[\left(\frac{w_t \alpha}{(1-\alpha)\chi R}\right) \left(\left(\frac{\chi(1-\alpha)R}{\alpha w_t}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P_t^l}{P_t^k}\right)^\sigma + 1\right) L_t\right]^\eta \overline{L}^{1-\eta}} \\ &= A \left(\frac{L_t}{\overline{L}}\right)^{1-\eta}, \end{aligned}$$

where

$$A \equiv \frac{\left[P^k \varphi^\sigma \left(\frac{P^l}{P^k}\right)^\sigma + P^l\right] \left(\frac{w_t \alpha}{(1-\alpha)\chi R}\right)^{\alpha-\eta}}{\left[\left(\frac{\chi(1-\alpha)R}{\alpha \underline{w}}\right)^{1-\alpha} \varphi^\sigma \left(\frac{P^l}{P^k}\right)^\sigma + 1\right]^\alpha}.$$

The second equality is derived by substituting (1) into (35), the third equality comes from (6), and the last equality holds because $w_t = \underline{w}$ during the transition. It follows that

$$\begin{aligned} \frac{TFP_{t+1}}{TFP_t} &= \left(\frac{L_{t+1}}{L_t}\right)^{1-\eta} \\ &= g_A^{1-\eta}, \end{aligned}$$

where the second equality follows from Proposition 2.

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