DISCOUNT SHOCK, PRICE–RENT DYNAMICS, AND THE BUSINESS CYCLE*

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The price-rent ratio in commercial real estate is highly volatile and its variation comoves with the business cycle. To account for these facts, we develop a dynamic general equilibrium model that introduces a rental market and incorporates the liquidity constraint on an individual firm's production as a key ingredient. Our estimation identifies the discount shock as the most important factor in driving price-rent dynamics and linking the dynamics in the real estate market to those in the real economy. We illustrate the importance of the liquidity premium and endogenous TFP in the nexus of the financial and real sectors.

1. INTRODUCTION

The rise and fall of real estate prices in the past decades and the 2008 financial crisis triggered by the collapse of real estate prices have generated a great deal of research on the impact of real estate prices on the macroeconomy. Most research has focused on consumers' behavior and the residential real estate market. When we study firms' investment dynamics, it is often the commercial real estate market that becomes relevant. In a recent paper, Chaney et al. (2012) provide micro evidence that links the commercial real estate price to investment. They estimate that a $1 increase in a representative U.S. firm's value of real estate raises its investment by $0.06. At the aggregate level, however, the link between commercial real estate prices and investment dynamics has been largely unexplored.

In this article, we develop a medium-size dynamic stochastic general equilibrium (DSGE) model and show that this model is capable of reproducing quantitatively key stylized facts about the commercial real estate price and the business cycle if one incorporates two key ingredients: shocks to households' subjective discount rate and the liquidity constraint on an individual firm's production. We call these shocks "discount shocks." We confront our model with financial and real time series and estimate it using the Bayesian method of Fernández-Villaverde and Rubio-Ramírez (2007) and Herbst and Schorfheide (2015) to account for the following two salient facts:

FIGURE 1

   i. *Volatility*: Commercial real estate price fluctuates much more than rent. Over the past 20 years, while the volatility (measured by the standard deviation of quarterly changes) is about 1% for real estate rent, the volatility of real estate prices is 4%.
  ii. *Comovements*: The price–rent ratio comoves with output as demonstrated by Figure 1. Since consumption and investment comove with output, the price–rent ratio tends to also move together with consumption and investment.

   How to account for these facts within one structural framework has been a challenging task in the macro-finance literature. The existing general equilibrium models with real estate markets typically fail to generate large price–rent variations.[1] Our model builds on the DSGE literature with a combination of two distinctive features: we introduce a rental market of commercial real estate and assume that an individual firm faces a liquidity constraint when financing its working capital. Without modeling the rental market explicitly, the existing macroeconomic models (Iacoviello, 2005; Iacoviello and Neri, 2010; Liu et al., 2013, 2016, for example) reveal that the real estate price and rent move in comparable magnitude so that there is little price–rent variation, which is inconsistent with what is observed in the data (Figure 1). As a result,

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[1] See Campbell et al. (2009), Piazzesi and Schneider (2009), Kiyotaki et al. (2011), Caplin and Leahy (2011), Burnside et al. (2011), Pintus and Wen (2013), Head et al. (2014), and Kaplan et al. (Forthcoming) for models of housing. This literature does not address the commercial housing market nor does it reproduce facts (i) and (ii) simultaneously in one dynamic general equilibrium framework.

traditional business-cycle shocks, such as shocks to technology and labor supply, cannot explain price–rent movements that are quantitatively comparable to the observed time series.

By controlling for an array of commonly used shocks such as technology and labor supply shocks, we find that shocks to the discount rate are the key to generating the data dynamics that account for stylized facts (i) and (ii) simultaneously. The key intuition is that the rental price of commercial real estate is determined by the marginal product of the real estate property in firms' production, but the real estate price is a forward looking variable, equal to the discounted present value of future rents and future liquidity premia for firms' production:

$$(1) \qquad p_t = E_t M_{t+1}(R_{ct+1} + p_{t+1}) + E_t M_{t+1} p_{t+1}^{\ell},$$

where p_t is the real estate price at time t, R_{ct} is the rental value of the real estate property, M_t is a stochastic discount factor (SDF), and p_t^{ℓ} is the liquidity premium that captures the impact of liquidity constraints on firms' production decisions. The standard asset pricing equation has only the first term on the right side of Equation (1). As shown in this article, however, it is the future liquidity premium (the second term on the right side of Equation (1)) that directly moves the current real estate price but not the current rent; and it is the shock to the SDF M_{t+1} that drives the fluctuation of the future liquidity premium.

The discount shock is a parsimonious way of modeling the variation in discount rates stressed by Hansen and Jagannathan (1991), Campbell and Ammer (1993), and Cochrane (2011) and can sometimes be interpreted as a sentiment shock as in Barberis et al. (1998) and Dumas et al. (2009). In the macroeconomic literature (Smets and Wouters, 2003; Galí, 2015, for example), the discount shock is called a "preference" shock to capture shifts in aggregate demand; its asset pricing implications were first discussed by Albuquerque et al. (2016), who construct a general equilibrium model of an endowment economy to show that discount shocks can generate the observed risk premium and weak correlation between consumption growth and stock returns. In their model, therefore, these shocks do not affect macroeconomic movements. Hall (2017) shows that discount shocks are most important in explaining the employment dynamics but does not explore asset pricing implications.

We construct a dynamic general equilibrium model that tightly links the financial market and the real economy by introducing discount shocks and the endogenous total factor productivity (TFP) into the model with a special emphasis on price–rent dynamics and the business cycle.[2] One key contribution of our article is to show that in our production economy model, it is the model's internal transmission mechanism that amplifies this small shock into large price–rent fluctuations over the business cycle. Because the constraint on firms' cash flow affects firms' borrowing capacity, both an individual firm's real estate value and the liquidity premium in the financial market play an essential role in expanding the firm's borrowing capacity for its current production as well as a flow of its future dividends (cash flow). We show how the nexus of the real estate price and the liquidity premium, mainly driven by discount shocks, accounts for large price–rent dynamics and their relationship to the business cycle.

The rest of the article is organized as follows: In Section 2, we construct a medium-size general equilibrium model with a production economy. In Section 3, we estimate the model against several U.S. time series, report the estimated results, analyze the impulse responses, discuss the linkage between price–rent dynamics and aggregate fluctuations, and provide key theoretical results. Section 4 concludes the article. Detailed derivations, proofs, and estimation procedures are provided in appendices.

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[2] Liu and Wang (2014) study a similar mechanism in a model with credit constraints and heterogeneous firms. Their model, however, does not include the real estate sector.

2.   THE MODEL

We study an economy with a representative household, a continuum of intermediate-goods producers, and a continuum of heterogeneous final-goods firms. The representative household maximizes its utility and accumulates physical capital. There are a variety of intermediate goods and each good is produced by a continuum of identical competitive producers. The heterogeneous final-goods firms are indexed by idiosyncratic productivity shocks. They trade commercial real estate properties among themselves and rent out real estate properties to intermediate-goods producers. Financial frictions occur in the final-goods sector; firms in this sector use unsecured credit to finance working capital.

2.1. *Households.*   The representative household maximizes the expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \Theta_t \beta^t \left[\log\left(C_t - \gamma C_{t-1}\right) - \psi_t \frac{N_t^{1+\nu}}{1+\nu} \right],$$

where C_t and N_t represent consumption and labor supply. The parameters $\beta \in (0,1)$ and $\gamma \in (0,1)$ represent the subjective discount factor and the household's habit formation. The variables $\theta_t \equiv \Theta_t/\Theta_{t-1}$ and ψ_t are exogenous shocks to the discount rate and labor supply; they follow an AR(1) process as

(2) $$\log \theta_t = (1 - \rho_\theta) \log \theta + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta,t},$$

(3) $$\log \psi_t = (1 - \rho_\psi) \log \psi + \rho_\psi \log \psi_{t-1} + \sigma_\psi \varepsilon_{\psi,t},$$

where $\varepsilon_{\theta,t}$ and $\varepsilon_{\psi,t}$ are i.i.d. standard normal random variables. Albuquerque et al. (2016) introduce discount shocks like ours as demand shocks in their endowment economy to study asset pricing implications.[3] In our model with a production economy, we examine empirically the importance of discount shocks in linking price–rent dynamics to the business cycle.

The household chooses consumption C_t, investment I_t, the capital utilization rate u_t, and risk-free bonds B_{t+1} subject to intertemporal budget constraint

$$C_t + \frac{I_t}{Z_t} + \frac{B_{t+1}}{R_{ft}} \le w_t N_t + R_{kt}(u_t K_t) + D_t + B_t,$$

where K_t, w_t, D_t, R_{kt}, and R_{ft} represent, respectively, capital, wage, dividend income, the rental price of capital, and the risk-free interest rate. The variable Z_t represents an aggregate investment-specific technology shock that has both permanent and transitory components (Greenwood et al., 1997; Krusell et al., 2000):

$$Z_t = Z_t^p v_{zt}, \ Z_t^p = Z_{t-1}^p g_{zt},$$

(4) $$\log g_{zt} = (1 - \rho_z) \log g_z + \rho_z \log(g_{z,t-1}) + \sigma_z \varepsilon_{zt},$$

(5) $$\log v_{zt} = \rho_{v_z} \log v_{z,t-1} + \sigma_{v_z} \varepsilon_{v_z,t},$$

where $\varepsilon_{z,t}$ and $\varepsilon_{v_z,t}$ are i.i.d. standard normal random variables.

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[3] Preference shocks used by Galí (2015) and other macroeconomic models relate to the log level of Θ_t. The shock process of $\log \theta_t$ relates to the discount factor β directly. We call it a discount shock.

Investment is subject to quadratic adjustment costs (Christiano et al., 2005). Capital evolves according to the law of motion

$$K_{t+1} = (1 - \delta_t)K_t + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_I\right)^2\right]I_t,$$

where $\delta_t \equiv \delta(u_t)$ is the capital depreciation rate in period t, g_I denotes the steady-state growth rate of investment, and Ω is the investment adjustment cost parameter.

2.2. *Intermediate-Goods Producers.* There is a continuum of intermediate goods. Each intermediate good $j \in [0, 1]$ is produced by a continuum of identical competitive producers of measure unity. The representative producer owns a constant-returns-to-scale technology to produce good j by hiring labor $N_t(j)$, renting real estate property $H_t(j)$ from final-goods firms, and renting capital $K_t(j)$ from the household. The producer's decision problem is

$$\max_{N_t(j),H_t(j),K_t(j)} P_{Xt}(j)X_t(j) - w_t N_t(j) - R_{ct}H_t(j) - R_{kt}K_t(j),$$

where $X_t(j) \equiv A_t[K_t^{1-\phi}(j)H_t^{\phi}(j)]^{\alpha}N_t^{1-\alpha}(j)$, $P_{Xt}(j)$ represents the competitive price of good j, and R_{ct} is the rental price of commercial real estate. The aggregate neutral technology shock A_t consists of both permanent and transitory components (Aguiar and Gopinath, 2007):

$$A_t = A_t^p v_{a,t}, \ A_t^p = A_{t-1}^p g_{at},$$

(6)
$$\log g_{at} = (1 - \rho_a)\log g_a + \rho_a \log(g_{a,t-1}) + \sigma_a \varepsilon_{at},$$

(7)
$$\log v_{a,t} = \rho_{v_a} \log v_{a,t-1} + \sigma_{v_a} \varepsilon_{v_a,t},$$

where ε_{at} and $\varepsilon_{v_a,t}$ are i.i.d. standard normal random variables.

2.3. *Final-Goods Firms.* There is a continuum of heterogeneous competitive firms. Each firm $i \in [0, 1]$ combines intermediate goods $x_t^i(j)$ to produce final consumption goods with the standard aggregation technology

(8)
$$y_t^i = a_t^i \exp\left(\int_0^1 \log x_t^i(j)dj\right),$$

where a_t^i represents an idiosyncratic productivity shock drawn independently and identically from a fixed distribution with probability density function (pdf) $f(a)$ and cumulative density function (cdf) $F(a)$ on the $(0, \infty)$ support. Firm i purchases intermediate good j at the price $P_{Xt}(j)$. The total spending on working capital is $\int_0^1 P_{Xt}(j)x_t^i(j)dj$. We assume that revenues arrive *after* working capital is utilized (a liquidity mismatch). Thus, firms must borrow to finance working capital.

Azariadis et al. (2016) document that unsecured credit has been far more important than secured credit for U.S. nonfinancial firms and Lian and Ma (2018) show that a vast majority (80%) of U.S. firms' debt is based on cash flow. In light of these facts, we follow closely Azariadis et al. (2016) by assuming that the firm finances working capital in the form of unsecured credit.

In each period t, prior to sales of output and real estate, firm i must borrow to finance its input costs. Intermediate-goods producers extend unsecured credit to the firm at the beginning of period t and allow it to pay input costs at the end of the period using revenues from sales of output and housing. The firm has limited commitment and may default on the unsecured credit. In the event of default, the firm would retain its production income y_t^i as well as its real estate holdings h_t^i. But the firm would be denied access to financial markets in the future. In particular, it would be barred from selling any asset holdings for profit and from obtaining loans for working capital. The following incentive compatibility constraint, similar to Azariadis et al. (2016), is imposed on the firm's optimization problem to make the contract self-enforceable:[4]

$$
(9) \qquad V_t(h_t^i, a_t^i) \geq (y_t^i + R_{ct} h_t^i) + E_t M_{t+1} V_{t+1}^a(h_t^i), \text{ all } t,
$$

where $V_t(h_t^i, a_t^i)$ denotes the firm's continuation value without default, $V_{t+1}^a(h_t^i)$ denotes the firm's continuation value in the default state. After default in period t, the firm's real estate holdings equal h_t^i forever. Here, the SDF M_{t+1} satisfies $M_{t+1} = \beta \Lambda_{t+1}/\Lambda_t$, where Λ_t denotes the marginal utility of consumption

$$
\Lambda_t = \frac{\Theta_t}{C_t - \gamma C_{t-1}} - \beta \gamma E_t \frac{\Theta_{t+1}}{C_{t+1} - \gamma C_t}.
$$

Constraint (9) is *not* always binding: whether a particular firm's credit constraint binds depends on the realization of its own productivity a_t^i.

Since $V_{t+1}^a(h_t^i)$ is equal to the sum of the rental value in period $t+1$ and the expected discounted present value of future rents, we have

$$
(10) \qquad E_t M_{t+1} V_{t+1}^a(h_t^i) = p_t^a h_t^i,
$$

where p_t^a denotes the expected discounted present value of future rents (per real estate unit)

$$
p_t^a \equiv E_t \sum_{\tau=1}^{\infty} M_{t+\tau} R_{ct+\tau} = E_t M_{t+1} (p_{t+1}^a + R_{ct+1}).
$$

In Appendix A.1, we show that the firm's expected continuation value without default satisfies

$$
(11) \qquad E_t M_{t+1} V_{t+1}(h_{t+1}^i, a_{t+1}^i) = p_t h_{t+1}^i.
$$

Following Azariadis et al. (2016), we call the difference between the expected continuation values per unit of real estate without default and with default the reputation value of the firm:

$$
b_t \equiv E_t M_{t+1} \big[V_{t+1}(h_{t+1}^i, a_{t+1}^i)/h_{t+1}^i - V_{t+1}^a(h_t^i)/h_t^i \big].
$$

By (10) and (11), the reputation value satisfies $b_t = p_t - p_t^a$. As argued by Azariadis et al. (2016), unsecured credit rests on the value that borrowers attach to a good credit reputation which is a forward-looking variable.

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[4] In an earlier version of this article (Miao et al., 2014), we use the standard collateral constraint in the model, where the collateral is the real estate. The current model has a much better fit to the data than the model with the standard collateral constraint. We follow Fernández-Villaverde and Rubio-Ramírez (2007) and compute log marginal data density (MDD) for each of the two models. Log MDD for the current model (1, 538.8) is larger than log MDD of Miao et al. (2014)'s model (1, 283.5) by more than 250. As shown in Subsection 3.3, moreover, the current model produces the comovement of the price–rent ratio with consumption in response to a discount shock, whereas the model with the standard collateral constraint fails in this important aspect.

Firm i trades real estate properties and rents some of them to the producers. The firm's income comes from profits and rents; its flow-of-funds constraint is given by

$$(12) \qquad d_t^i + p_t \left(h_{t+1}^i - h_t^i \right) = y_t^i - \int_0^1 P_{Xt}(j) x_t^i(j) dj + R_{ct} h_t^i,$$

where d_t^i denotes dividend and the initial condition h_0^i is given. Subject to (8), (9), (10), and (12), firm i's problem is to solve the Bellman equation

$$(13) \qquad V_t\left(h_t^i, a_t^i \right) = \max_{x_t^i(j), h_{t+1}^i \geq 0} \qquad d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(h_{t+1}^i, a_{t+1}^i \right).$$

Problem (13) is equivalent to maximizing the discounted present value of future dividends (cash flow). Thus, the credit constraint studied here is based on the present value of cash flow from the firm's continuing operations.

2.4. *Equilibrium.* The markets clear in real estate, government bond, and intermediate-goods sectors:

$$\int_0^1 h_t^i di = \int_0^1 H_t(j) dj = 1, \ B_t = 0, \ \int_0^1 x_t^i(j) di = X_t(j) = A_t \left[K_t^{1-\phi}(j) H_t^\phi(j) \right]^\alpha N_t^{1-\alpha}(j).$$

Since the equilibrium is symmetric across intermediate-goods producers, we have

$$P_{Xt}(j) = P_{Xt}, \ H_t(j) = H_t, \ N_t(j) = N_t, \ K_t(j) = u_t K_t,$$

$$X_t(j) = X_t = A_t \left[(u_t K_t)^{1-\phi} H_t^\phi \right]^\alpha N_t^{1-\alpha}$$

for all j. The household's dividend income and aggregate output are

$$D_t = \int_0^1 d_t^i di \quad \text{and} \quad Y_t = \int_0^1 y_t^i di.$$

The competitive equilibrium consists of price sequences $\{w_t, R_{ct}, R_{kt}, p_t, R_{ft}, P_{Xt}\}_{t=0}^\infty$ and allocation sequences $\{C_t, I_t, u_t, N_t, Y_t, B_{t+1}, K_{t+1}, X_t, D_t\}_{t=0}^\infty$ such that (a) given the prices, the allocations solve the optimizing problems for households, intermediate-goods producers, and final-goods firms and (b) all markets clear.

3. ESTIMATION AND ANALYSIS

3.1. *Data and Estimation.* We take the Bayesian approach and estimate the log-linearized version of the model presented in Section 2. The model has six commonly used macroeconomic shocks represented by AR(1) processes (2), (3), (4), (5), (6), and (7). It is estimated against a number of key U.S. time series over the period from 1995Q2 to 2017Q2:[5] the commercial real estate price index, the commercial real estate rental index, the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), and per capita hours worked.[6] Since our model features long-run growth, we detrend our model to

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[5] The repeated sales price of commercial real estate is available from 1996Q2 until present. We allow four lags in estimation. Therefore, the sample including four lags begins in 1995Q2.

[6] In Chaney et al. (2012), commercial real estate prices are approximated by residential house prices because these two series are highly correlated (the correction is about 0.90). In this article, we use the time series of commercial real estate price directly.

Parameter	Representation	Posterior Estimates		
		Mode	Low	High
ν	Inv Frisch elasticity	0.497	0.154	1.065
η	Survival elasticity	3.252	3.065	3.619
δ''/δ'	Capacity utilization	5.422	3.328	12.44
γ	Habit formation	0.134	0.045	0.297
Ω	Capital adjustment	0.051	0.025	0.130

NOTE: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

make it stationary. We use $\tilde{x}_t$ to denote the detrended variable of x_t and use $\hat{x}_t \equiv \log \tilde{x}_t - \log \tilde{x}$ to denote the log deviation from the steady-state value $\tilde{x}$. The detailed description of data and estimation method are provided in Appendices A.2 and A.3.[7]

There are five structural parameters to be estimated: the inverse Frisch elasticity ν, the steady-state survival elasticity η for firms, the steady-state elasticity of capacity utilization δ''/δ', the habit formation γ, and the investment adjustment cost Ω. The survival elasticity η measures the degree of heterogeneity of firms: the larger the value is, the more important the endogenous TFP becomes (see Appendix A.4 for further discussions). The other structural parameters are either calibrated or indirectly estimated by solving the steady state.

The five directly estimated parameters are reported in Table 1, along with 90% probability intervals. The posterior probability intervals indicate that all these structural parameters are tightly estimated. The mode estimate of the inverse Frisch elasticity of labor supply is 0.497, consistent with a range of values discussed in the literature (Keane and Rogerson, 2011). The survival elasticity is tightly estimated around 3.252, implying the importance of the endogenous TFP in propagating the business cycle as discussed in Subsection 3.2.

The steady-state elasticity of capacity utilization δ''/δ' is 5.42. The high value means that an increase in the marginal cost is significant when capacity increases, which implies that capacity does not respond strongly to economic shocks. The estimated habit formation γ and capital-adjustment cost Ω are very small, implying that these factors are not important in driving the dynamics of consumption and investment.

Table 2 reports the estimated persistence and standard-deviation parameters of exogenous shock processes. Among all shocks, the discount shock is the most persistent. But its estimated standard deviation is considerably smaller than those of all other shocks except the stationary investment-specific shock. The probability intervals for the estimated standard deviation of the discount shock are particularly tight. Such a small standard deviation implies that any large effects on real estate price and aggregate variables must come from the model's internal propagation mechanism, which is discussed in Subsection 3.2.

3.2. *Propagation Mechanism.* A tractable feature of our heterogeneous model is that one can obtain a closed-form solution to the aggregation problem. The closed-form solution is essential to make our estimation and empirical analysis feasible. In Supplemental Appendix E, we list all the equilibrium equations for solving and estimating the model. In this section, we emphasize the key equilibrium dynamics and highlight the role of financial frictions in the transmission mechanism.

Denote the average cost of intermediate goods by

$$(14) \qquad a_t^* \equiv \exp\left[\int_0^1 \log P_{Xt}(j)dj\right] = P_{Xt}.$$

[7] The complete equilibrium system, the detrended stationary equilibrium system, the steady state, and the log-linearized equilibrium system are presented in Supplemental Appendices E, F, G, and H.

Parameter	Representation	Posterior Estimates		
		Mode	Low	High
ρ_z	Permanent investment tech	0.1285	0.0505	0.2623
ρ_{v_z}	Stationary investment tech	0.1285	0.0253	0.8670
ρ_a	Permanent neutral tech	0.1777	0.0222	0.5527
ρ_{v_a}	Stationary neutral tech	0.8588	0.7811	0.8920
ρ_θ	Discount rate	0.9993	0.9980	0.9998
ρ_ψ	Labor supply	0.9944	0.9780	0.9978
σ_z	Permanent investment tech	0.0052	0.0044	0.0059
σ_{v_z}	Stationary investment tech	0.0001	0.0001	0.0016
σ_a	Permanent neutral tech	0.0040	0.0022	0.0056
σ_{v_a}	Stationary neutral tech	0.0104	0.0093	0.0125
σ_θ	Discount rate	0.0003	0.0003	0.0004
σ_ψ	Labor supply	0.0069	0.0053	0.0102

NOTE: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

The following two key propositions establish the close link between asset prices and the production economy:

PROPOSITION 1. *The optimal output for firm i is given by*

$$(15) \qquad y_t^i = \begin{cases} \frac{a_t^i}{a_t^*} b_t h_t^i & \text{if } a_t^i \geq a_t^*, \\ 0 & \text{otherwise,} \end{cases}$$

where the average cost a_t^* and aggregate output Y_t are determined jointly by the two simultaneous equations:

$$(16) \qquad \frac{b_t}{a_t^*} \int_{a_t^*}^{\infty} a f(a)\,da = Y_t,$$

and

$$(17) \qquad Y_t = A_t (u_t K_t)^{\alpha(1-\phi)} H_t^{\alpha\phi} N_t^{1-\alpha} \left[\frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} a f(a)\,da \right],$$

where the term in square brackets is the endogenously determined TFP.

PROOF. See Appendix A.1.1.

Proposition 1 states that the average cost of intermediate goods, a_t^*, is also a threshold productivity level, above which productive firms choose to produce. For a given value of Y_t, Equation (16) describes the relationship between the threshold productivity a_t^* and the reputational value b_t for the real estate price. This relationship is represented by an upward sloping curve on the (a_t^*, b_t) graph (bottom panel of Figure 2). Since Equation (16) is derived from the liquidity (cash flow) constraint, we call this relationship the liquidity constraint curve.

PROPOSITION 2. *The liquidity premium p_t^ℓ, the reputation value b_t, and the rent R_{ct} satisfy*

$$(18) \qquad p_t^\ell = b_t \int_{a_t^*}^{\infty} \frac{a - a_t^*}{a_t^*} f(a)\,da,$$
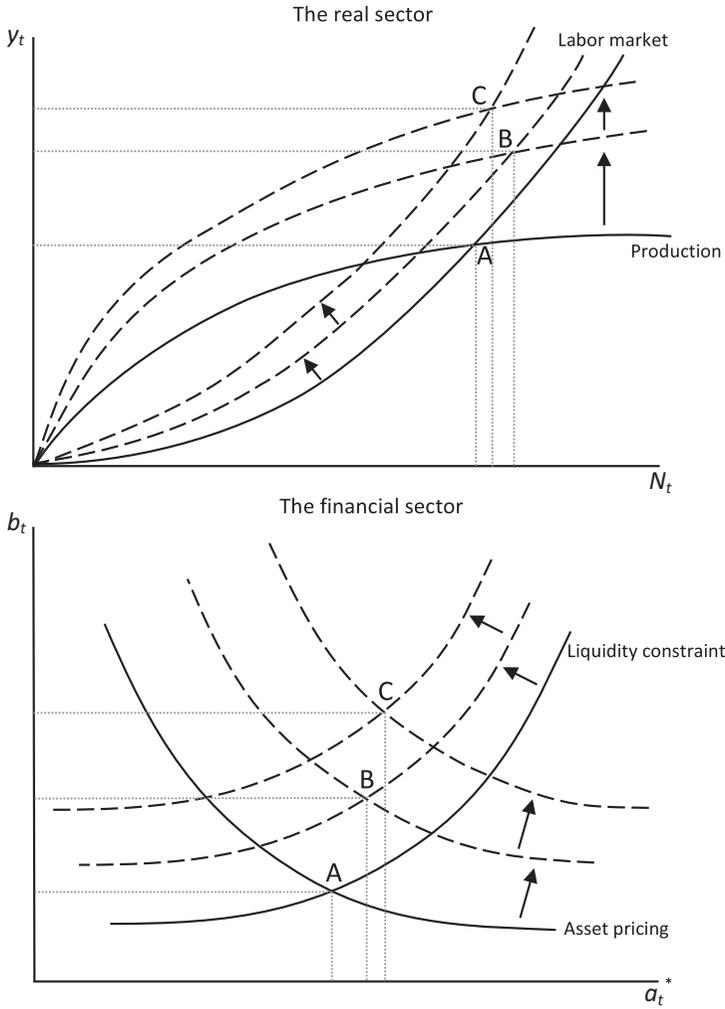
The real sector

The financial sector

FIGURE 2

$$(19) \qquad b_t = E_t M_{t+1} b_{t+1} \left[1 + \int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da \right],$$

$$(20) \qquad R_{ct} = \alpha \phi a_t^* A_t (u_t K_t)^{\alpha(1-\phi)} H_t^{\alpha\phi-1} N_t^{1-\alpha}.$$

PROOF.   See Appendix A.1.2.

The asset pricing equation (1) discussed in the introduction departs from the standard one in that the SDF and rent are not the only factors moving the real estate price. The standard asset pricing equation

$$p_t = E_t M_{t+1}(R_{ct+1} + p_{t+1})$$

misses the second term on the right side of (1). Proposition 2 states that, in addition to the future rent, the future liquidity premium represented by Equation (18) also influences the real estate price. For productive firms ($a_{t+1}^i \geq a_{t+1}^*$), the liquidity premium reflects the average profit generated by one dollar of unsecured credit. As shown in Subsection 3.3, the liquidity premium, not the rent, is a driving force of the fluctuation of the real estate price.

Equation (20) in Proposition 2 shows that the discount shock does not affect the current rent R_{ct} directly. It has an indirect effect through its impact on other variables such as N_t and a_t^*. On the other hand, the discount shock has a direct effect on the expected appreciation of future prices through its impact on the SDF M_{t+1} in both terms on the right side of Equation (1). Consequently, the discount shock has the potential to explain the dynamics of the price–rent ratio.

Substituting Equation (18) for p_{t+1}^ℓ in Equation (1), we obtain the complete asset pricing equation as

$$(21) \qquad p_t = E_t M_{t+1} \left[R_{ct+1} + p_{t+1} + b_{t+1} \int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*} f(a) da \right].$$

The relationship between a_t^* and b_t, represented by Equation (21), is negative, holding everything else fixed. An increase in the current threshold productivity level a_t^* raises the future threshold productivity level a_{t+1}^*. As a_{t+1}^* rises, one can see from Equation (19) that the future reputation value b_{t+1} falls. Thus, the asset pricing curve representing Equation (21) is downward sloping on the (a_t^*, b_t) plane. The two curves, liquidity constraint and asset pricing, determine a_t^* and b_t jointly in the financial market as plotted in the bottom panel of Figure 2.

To make transparent the connection between the real estate market and the production economy, one should note that the real wage and labor hours are jointly determined by the labor supply equation

$$\frac{\Lambda_t}{\Theta_t} w_t = \psi_t N_t^\nu$$

and the labor demand equation

$$(1 - \alpha) Y_t = \frac{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da}{1 - F(a_t^*)} w_t N_t.$$

Using these two equations to eliminate w_t, we obtain the equilibrium equation that determines labor hours:

$$(22) \qquad N_t^{1+\nu} = \frac{1 - F(a_t^*)}{\int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da} \frac{(1 - \alpha) Y_t \frac{\Lambda_t}{\Theta_t}}{\psi_t}.$$

The top panel of Figure 2 plots two curves on the (N_t, Y_t) graph, with the convex curve representing the production equation (17) and the concave curve representing the labor-market equation (22).

A discount shock affects both real and financial sectors simultaneously. Figure 2 illustrates the propagation of this shock. Suppose that the initial equilibrium is Point A at the steady state. According to Equation (1), a positive shock to the discount rate delivers a direct impact on the real estate price through the SDF M_{t+1}, shifting the asset pricing curve upward and raising the threshold productivity. As

$$TFP_t \equiv \frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} a f(a) da$$

is driven by the response of a_t^* to the discount shock, a rise of the threshold productivity increases aggregate output through the endogenous TFP and thus demand for investment and credit to finance working capital. An increase of aggregate output shifts the liquidity constraint curve upward according to Equation (16). The direct effect of the discount shock on asset prices dominates the indirect effect on aggregate output so that the net effect on the threshold productivity is positive (bottom panel of Figure 2). The equilibrium moves from Point A to Point B in the short run, with an increase of both threshold productivity and reputation value for the real estate price.

As an increase of the threshold productivity raises aggregate output and shifts the production curve upward, it simultaneously shifts the labor-market curve upward as long as the endogenous TFP relative to the average cost a_t^*

$$\frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} \frac{a}{a_t^*} f(a) da$$

increases with a_t^* and its large impact on consumption (and its marginal utility Λ_t) is postponed.

With capital accumulation, it is optimal for households to postpone consumption for investment. Thus, the hump-shaped response of investment propels a further increase of aggregate output and thus shifts the asset pricing curve further during subsequent periods. As a result of higher investment and output, the liquidity constraint curve moves up further, generating an even higher reputation value for the real estate price. As long as the discount shock is persistent as in our estimation, both the asset pricing curve and the liquidity constraint curve continue to shift upward, moving the equilibrium from Point B to Point C (bottom panel of Figure 2), with a persistent increase in the real estate price. In equilibrium, however, the threshold productivity level a_t^* does not have to move much as shown in the figure.

At the same time, a higher level of investment continues to shift the production curve and the labor-market curve upward, moving the equilibrium from Point B to Point C (top panel of Figure 2) and generating even higher output. The ripple effect through such interactions between the financial sector and the production sector, the key feature of this propagation mechanism, is able to generate the long and large hump-shape response of both consumption and output, even though the discount shock itself has no hump shape and the magnitude of the volatility is extremely small (Table 2), as will be further discussed in the next section.

3.3. *Impulse Responses to Discount and Technology Shocks.*   The preceding section explains the propagation mechanism for the linkage between the financial and production sectors. In this section, we document both the financial and real impacts of a discount shock *after* controlling for all other common shocks studied in the literature. Among other common shocks, neutral technology shocks are most important in driving the business cycle. We thus compare the estimated dynamic responses to a discount shock with those to a permanent technology shock in Figures 3 and 4. Although the discount shock process is assumed to be of AR(1) and the estimated habit and capital-adjustment cost are extremely small, the discount shock generates sizable hump-shaped responses of consumption and investment in magnitude comparable to the dynamic responses to the technology shock. As explained in Subsection 3.2, it is the model's endogenous propagation mechanism that generates such a hump shape.

The dynamic responses of labor hours to discount and technology shocks are also humped, but due to the wealth effect of the permanent nature of the technology shock, the magnitude of the labor hours response to this shock is very small in comparison to the effects of the discount shock. The importance of a discount shock in the business cycle is consistent with the finding of Hall (2017) that such a shock plays a significant role in the fluctuation of the labor market. Indeed, our variance decomposition reveals that the discount rate explains 43% of the hours fluctuation at the five-year forecast horizon.[8]
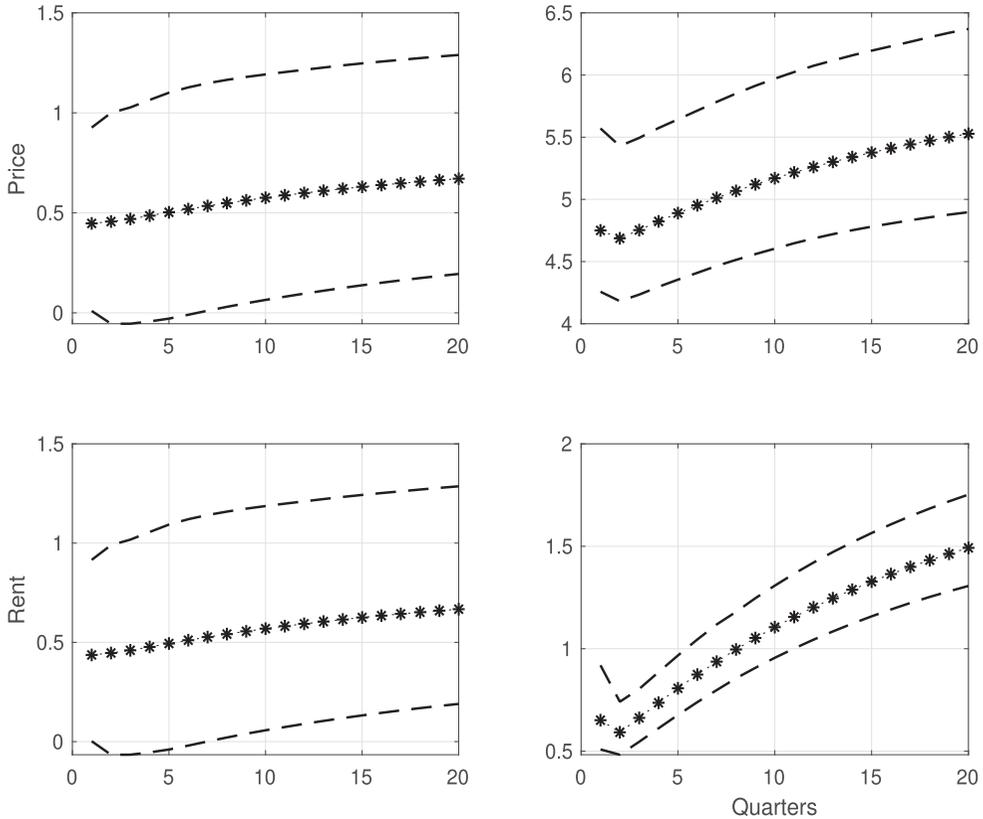
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[8] The discount rate explains 60% of the consumption fluctuation and 66% of the investment fluctuation.
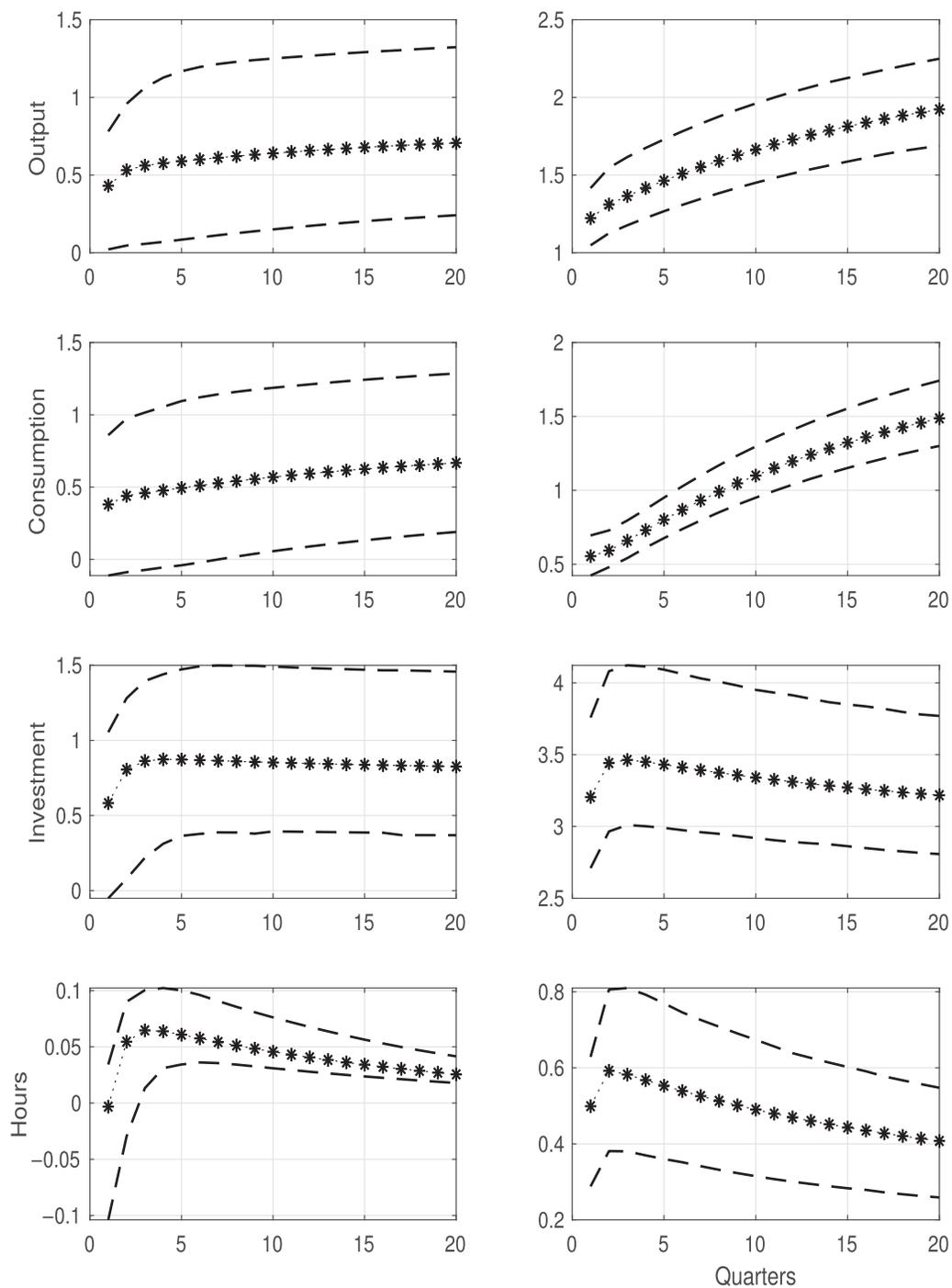
NOTE: The starred line represents the estimated response. The dashed lines represent the 0.90 probability error bands.

FIGURE 3

IMPULSE RESPONSES (%) TO A ONE-STANDARD-DEVIATION SHOCK TO NEUTRAL TECHNOLOGY GROWTH (LEFT PANEL) AND TO DISCOUNT RATES (RIGHT PANEL)

FIGURE 4

IMPULSE RESPONSES (%) TO A ONE-STANDARD-DEVIATION SHOCK TO NEUTRAL TECHNOLOGY GROWTH (LEFT PANEL) AND TO DISCOUNT RATES (RIGHT PANEL). THE STARRED LINE REPRESENTS THE ESTIMATED RESPONSE

Sharper differences between these two shocks show up in the dynamic responses of real estate price and rent. To a technology shock, the dynamic responses of price and rent move almost in the same magnitude (left column of Figure 4). The technology shock thus generates little movement of the price–rent ratio. In contrast, the price response to a discount shock is considerably larger than the rent response (right column of Figure 4). Since price fluctuates much more than rent in the data, it is the discount shock, not the technology shock, that can explain almost all the price–rent fluctuation.[9] According to the variance decomposition, about 94% at the five-year forecast horizon is explained by the discount shock.[10]

As discussed in Subsection 3.2, the impact of a discount shock works through the future SDF M_{t+1}, which in turn directly affects the fluctuation of the risk-free rate $\log R_{ft}$. The issue then is whether a discount shock would generate too much of the volatility of $\log R_{ft}$ in comparison with

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[9] If one takes into account nonlinearity in the model as shown by Fernández-Villaverde and Rubio-Ramírez (2005), other shocks may matter but only in a second order.

[10] To be sure, the self-enforcing incentive compatibility constraint is not the only mechanism that can account for the price–rent fluctuation. Any mechanism that helps move the liquidity premium in the asset pricing equation, represented by (1), has the potential to explain the large fluctuation of the observed price–rent ratio. In the bubble literature (Miao et al., 2015; Miao and Wang, 2018), for example, sentiment shocks through the asset bubble mechanism can generate the liquidity premium in the asset pricing equation and thus are capable of accounting for the large price–rent fluctuation. Another example is sunspot shocks studied by Azariadis et al. (2016) through the multiple equilibria mechanism. In short, asset bubbles and the reputation value, driven by sentiment or sunspot shocks, are examples of other mechanisms that can account for the large price–rent fluctuation.

Description	Volatility	Data	Model	
			Low	High
Real estate price	$std(\Delta \log p_t)$	4.171	3.542	4.560
Rental price	$std(\Delta \log R_{ct})$	1.245	0.475	0.635
Price–rent ratio	$std(\Delta \log(p_t/R_{ct}))$	3.909	3.085	3.971
Risk-free rate	$std(\log R_{ft})$	2.135	0.123	0.244
Discount rates	$std(\log \theta_t)$		0.053	0.183

NOTE: "Low" and "High" denote the bounds of the 90% probability interval of the simulated data from the model.
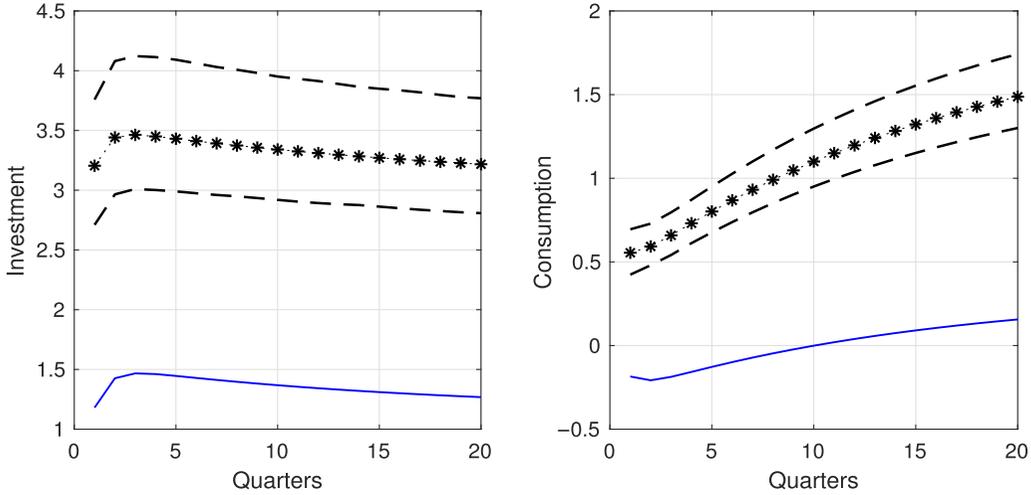
the data. We compute the quarterly volatility of the three-month Treasury bill rate for the same period as the sample period used for our model. As reported in Table 3, the quarterly volatility in the data is 2.135%. We then compare it to the volatility implied by the model-simulated data. Based on the model parameter estimates, we simulate a sample of time series of all variables with only discount shocks for 88 periods (the same length as the actual sample size). For each simulated sample, we compute the volatility of $\log R_{ft}$. We repeat the simulation 100,000 times and compute the 90% probability interval of this volatility, which is between 0.12% and 0.24%. The simulation result shows that discount shocks in our model do not generate an unreasonably large volatility of $\log R_{ft}$ in comparison with the data.

Despite the high persistence of the discount shock, the estimated volatility is extremely small (Table 3). Yet, one can see from Table 3 that the discount shock can generate much larger volatilities of the real estate price and the price–rent ratio in magnitude comparable to the data while generating a much smaller volatility of the real estate rent than the actual volatility. The 90% probability intervals for the volatilities of the real estate price and the price–rent ratio contain the actual volatilities in the data, implying that the model dynamics are reasonable. The driving force of these large volatilities, as discussed in Subsection 3.2, is the propagation mechanism of the liquidity premium for both the real estate price and highly productive firms that tend to be constrained by cash flow.

Not only is the discount shock important in driving the movements of the real estate variables, but it can also generate significant comovements among the price–rent ratio, consumption, investment, hours, and output, as presented in the right column of Figures 3 and 4. In particular, consumption comoves with the price–rent ratio in response to a discount shock (Figures 4 and 5). The comovement is due to the endogenous TFP channel discussed in Subsection 3.2. A positive discount shock raises endogenous TFP and hence output, which allows consumption and investment to rise at the same time. Without financial frictions, by contrast, the consumption response to a discount shock is negative for the initial 10 quarters whereas the response of investment and hours remain positive (solid lines in Figure 5). Thus, the two ingredients commonly used in the modern macroeconomic literature, credit constraint and endogenous TFP, are essential to mitigating the opposite movements between consumption and investment responses as well as to generating a strong response of the real estate price.

4. CONCLUSION

We argue that imbedding households' discount shocks and firms' liquidity constraints in the dynamic general equilibrium framework can substantially improve the model's performance in accounting for the large volatility of the price–rent ratio in the real estate market and the comovements among the price–rent ratio, consumption, investment, and hours. We find that the liquidity premium is the most important factor in linking financial and real variables. The volatility of the estimated discount shocks is extremely small in magnitude but the model's

FIGURE 5

IMPULSE RESPONSES (%) TO A ONE-STANDARD-DEVIATION SHOCK TO DISCOUNT RATES [COLOR FIGURE CAN BE VIEWED AT WILEYONLINELIBRARY.COM]

internal propagation mechanism translates these small shocks into the large volatilities of the price–rent ratio and the business cycle.

Because the 2008 financial crisis was triggered by the collapse of real estate prices and the sharp fall of investment, this article takes a step to focus on the commercial real estate and its relation to the business cycle by illustrating an economic mechanism that moves the liquidity premium in the asset pricing equation and accounts for the large fluctuation of the observed price–rent ratio. It abstracts from other dimensions that merit further study in the future. One such dimension is to include mortgage markets for households. Another dimension is to extend the model to incorporate the stock market in the model. We hope that the mechanism and insight developed in this article lays the groundwork for extending the model along these and other important dimensions.

APPENDIX

A.1. *Proposition Proofs.*

A.1.1. *Proof of proposition 1.*   We rewrite firm i's decision problem as the Bellman equation

$$(A.1) \qquad V_t(h_t^i, a_t^i) = \max_{x_t^i(j), h_{t+1}^i \geq 0} \quad d_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(h_{t+1}^i, a_{t+1}^i)$$

subject to (8), (9), and (12) .

To solve the firm's decision problem, we first derive the unit cost of production. Define the total cost of producing y_{it} as

$$\Phi(y_t^i, a_t^i) \equiv \min_{x_t^i(j)} \quad \int P_{Xt}(j) x_t^i(j) dj$$

subject to $a_t^i \exp(\int_0^1 \log x_t^i(j)\,dj) \geq y_t^i$. Cost minimization implies that

$$\text{(A.2)} \qquad \Phi(y_t^i, a_t^i) = y_t^i \frac{a_t^*}{a_t^i},$$

where the average cost a_t^* is given by Equation (14) and the demand for each $x_t^i(j)$ satisfies

$$\text{(A.3)} \qquad P_{Xt}(j)x_t^i(j) = a_t^* \exp\left(\int_0^1 \log x_t^i(j)\,dj\right).$$

Using the cost function in (A.2), we rewrite firm i's budget constraint as

$$\text{(A.4)} \qquad d_t^i + p_t(h_{t+1}^i - h_t^i) \leq y_t^i - y_t^i \frac{a_t^*}{a_t^i} + R_{ct}h_t^i.$$

Conjecture the value function in the form of

$$\text{(A.5)} \qquad V_t(h_t^i, a_t^i) = v_t(a_t^i)h_t^i.$$

By the Bellman equation, we have

$$v_t(a_t^i)h_t^i = \max_{y_t^i, h_{t+1}^i \geq 0} y_t^i\left(1 - \frac{a_t^*}{a_t^i}\right) + R_{ct}h_t^i$$
$$- p_t(h_{t+1}^i - h_t^i) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}(a_{t+1}^i)h_{t+1}^i.$$

If $p_t > \beta E_t[v_{t+1}(a_{t+1}^i)\Lambda_{t+1}/\Lambda_t]$, firm i would prefer to sell all real estate so that $h_{t+1}^i = 0$. All other firms would not hold real estate because the preceding inequality holds for any i as a_t^i is an i.i.d. process. This would violate the market clearing condition for the real estate market. If $p_t < \beta E_t[v_{t+1}(a_{t+1}^i)\Lambda_{t+1}/\Lambda_t]$, all firms would prefer to own real estate as much as possible, which again violates the market clearing condition. Thus, we have

$$\text{(A.6)} \qquad p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}(a_{t+1}^i).$$

Equation (A.6) is an equilibrium restriction on the real estate price.

Using the Bellman equation (13), we can rewrite the incentive constraint (9) as

$$\text{(A.7)} \qquad y_t^i\left(1 - \frac{a_t^*}{a_t^i}\right) + R_{ct}h_t^i - p_t(h_{t+1}^i - h_t^i) + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} v_{t+1}(a_{t+1}^i)h_{t+1}^i$$
$$\geq y_t^i + R_{ct}h_t^i + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^a(h_t^i).$$

Using (A.6), (10), and $b_t = p_t - p_t^a$, we can rewrite this constraint as

$$\text{(A.8)} \qquad y_t^i \frac{a_t^*}{a_t^i} \leq b_t h_t^i.$$

Substituting Equations (A.4) and (A.5) into Equation (A.1), we rewrite the firm's problem as

$$(A.9) \qquad v_t(a_t^i)h_t^i = \max_{y_t^i,\, h_{t+1}^i} y_t^i\left(1 - \frac{a_t^*}{a_t^i}\right) + R_{ct}h_t^i - p_t(h_{t+1}^i - h_t^i) + p_t h_{t+1}^i$$

subject to (A.8). The optimal solution to (A.9) is

$$(A.10) \qquad y_t^i = \begin{cases} \frac{a_t^i}{a_t^*}b_t h_t^i & \text{if } a_t^i \geq a_t^* \\ 0 & \text{otherwise.} \end{cases}$$

Aggregating individual firms' output in (A.10) gives

$$(A.11) \qquad Y_t = \int_0^1 y_t^i di = \int_{a_t^i \geq a_t^*}^{\infty} \frac{a_t^i}{a_t^*} b_t da_t^i \int_0^1 h_t^i di = \frac{b_t}{a_t^*}\int_{a_t^*}^{\infty} af(a)da.$$

From Equations (8), (A.3), and (A.7), one can see that the total production cost is given by

$$P_{Xt}X_t = \int_0^1 \int_0^1 P_{Xt}x_t^i(j)didj = \int_{a_t^i \geq a_t^*}^{\infty} \frac{a_t^*}{a_t^i}y_t^i di = b_t[1 - F(a_t^*)].$$

Using $P_{Xt} = a_t^*$ and $X_t(j) = X_t = A_t[(u_tK_t)^{1-\phi}H_t^\phi]^\alpha N_t^{1-\alpha}$, we derive

$$(A.12) \qquad b_t = \frac{a_t^* A_t\left[(u_tK_t)^{1-\phi}H_t^\phi\right]^\alpha N_t^{1-\alpha}}{1 - F(a_t^*)}.$$

Combining this equation and (A.11) gives the aggregate production function

$$Y_t = A_t\left[(u_tK_t)^{1-\phi}H_t^\phi\right]^\alpha N_t^{1-\alpha}\frac{\int_{a_t^*}^{\infty} af(a)da}{1 - F(a_t^*)}.$$

A.1.2. *Proof of proposition 2.* Substituting Equation (A.10) into the Bellman equation (A.9) and matching the coefficients, we obtain

$$v_t(a_t^i) = \begin{cases} \left(\frac{a_t^i}{a_t^*} - 1\right)b_t + R_{ct} + p_t & \text{if } a_t^i \geq a_t^* \\ R_{ct} + p_t & \text{otherwise.} \end{cases}$$

Substituting the above expression into (A.6) gives the asset pricing equation

$$p_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{ct+1} + p_{t+1} + b_{t+1}\int_{a_{t+1}^*}^{\infty} \frac{a - a_{t+1}^*}{a_{t+1}^*}f(a)da\right].$$

The first-order condition for the optimal problem of intermediate goods producers with respect to real estate gives

$$\alpha\phi P_{Xt}(j)A_t K_t(j)^{\alpha(1-\phi)}N_t(j)^{(1-\alpha)}H_t^{\alpha\phi-1}(j) = R_{ct}.$$

Given the symmetric equilibrium in the intermediate goods sector and the market clear conditions, the previous equation becomes

$$(A.13) \qquad R_{ct} = \alpha\phi a_t^* A_t(u_tK_t)^{\alpha(1-\phi)}H_t^{\alpha\phi-1}N_t^{1-\alpha}.$$

A.2. *Data.* All the quarterly time series used in this article were constructed by Patrick Higgins at the Federal Reserve Bank of Atlanta, some of which were collected directly from the Haver Analytics Database (Haver for short). In this section, we describe the details of data construction.

The model estimation is based on six U.S. aggregate time series: the real price of commercial real estate (p_t^{Data}), the real rental price (R_{ct}^{Data}), the quality-adjusted relative price of investment ($(1/Z_t)^{\text{Data}}$), real per capita consumption (C_t^{Data}), real per capita investment (I_t^{Data}), and per capita total hours (H_t^{Data}). All variables except hours and relative price of investment are deflated by the price of nondurable consumption goods and nonhousing services.

These series are constructed as follows:

- $p_t^{\text{Data}} = \frac{\text{pCommRE}}{\text{PriceNonDurPlusServExHous}}.$

- $R_{ct}^{\text{Data}} = \frac{\text{TortoTotalRent}}{\text{PriceNonDurPlusServExHous}}.$

- $(1/Z_t)^{\text{Data}} = \frac{\text{GordonPriceCDplusES}}{\text{PriceNonDurPlusServExHous}}.$

- $C_t^{\text{Data}} = \frac{(\text{NomConsNHSplusND})/\text{PriceNonDurPlusServExHous}}{\text{POPSMOOTH\_USECON}}.$

- $I_t^{\text{Data}} = \frac{(\text{CDX\_USNA}+\text{nominveqipp})/\text{PriceNonDurPlusServExHous}}{\text{POPSMOOTH\_USECON}}.$

- $H_t^{\text{Data}} = \frac{\text{AggHours}}{\text{POPSMOOTH\_USECON}}.$

Sources for the constructed data, along with the Haver keys (all capitalized letters) to the data, are described below.

pCommRE Commercial real-estate price index. The construction of this series is based on the series named as "FL075035503" from the Flow of Funds Accounts database provided by the Board of Governors of the Federal Reserve System.[11] Note that the price index through 1996Q1 is *not based on repeated sales* but instead relies on a weighted average of three appraisal-based commercial property price series (per square foot): retail property, office property, and warehouse/industrial property. These series come from the National Real Estate Investor (NREI). The weights applied to the NREI are not revised and are calculated using annual data from the Survey of Current Business. From 1996Q2 forward, the commercial property price index is the Costar Commercial Repeat Sales Index published by "National Real Estate Investor."

TortoTotalRent Rental price index for commercial real estate. Tornqvist aggregate of Torto Wheaton Research Index for rental prices of retail properties, Torto Wheaton Research Index for rental prices of office properties (commercial excluding retail), and Torto Wheaton Research Index for rental prices of industrial properties. Detailed description of the series is available at http://www.cohenasset.com/pdfs/Torto-Wheaton-Research-Methodology.pdf. The data, downloaded from the CBRE Econometrics Advisors Web site, were constructed by the Torto Wheaton Research (TWR) hedonic approach (Wheaton and Torto, 1994) and (Malpezzi, 2002, Chapter 5).

PriceNonDurPlusServExHous Consumer price index. Price deflator of nondurable consumption and nonhousing services, constructed by Tornqvist aggregation of price deflator of nondurable consumption and nonhousing related services (2009 = 100).

GordonPriceCDplusES Price of investment goods. Quality-adjusted price index for consumer durable goods, equipment investment, and intellectual property products investment. This is a weighted index from a number of individual price series within this category. For each individual price series from 1947 to 1983, we use Gordon (1990)'s quality-adjusted price

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[11] See the Federal Reserve Board of Governors' Web site http://www.federalreserve.gov/apps/fof/SeriesAnalyzer. aspx?s=FI075035503&t=. Unpublished tables for level time series are available at http://www.federalreserve.gov/ datadownload/Choose.aspx?rel=Z.1.

index. Following Cummins and Violante (2002), we estimate an econometric model of Gordon's price series as a function of time trend and several macroeconomic indicators in the National Income and Product Account (NIPA), including the current and lagged values of the corresponding NIPA price series. The estimated coefficients are then used to extrapolate the quality-adjusted price index for each individual price series for the sample from 1984 to 2008. These constructed price series are annual. We use Denton (1971)'s method to interpolate these annual series at quarterly frequency. We then use the Tornqvist procedure to construct the quality-adjusted price index from the interpolated individual quarterly price series.

NomConsNHSplusND Nominal personal consumption expenditures. Nominal nondurable goods and nonhousing services (SAAR, billions of dollars). It is computed as CNX_USNA + CSX_USNA - CSRUX_USNA, where CNX_USNA is nominal nondurable goods consumption (SAAR, millions of dollars), CSX_USNA is nominal service consumption (SAAR, millions of dollars), and CSRUX_USNA is nominal housing and utilities consumption (SAAR, millions of dollars).

POPSMOOTH_USECON Population. Smoothed civilian noninstitutional population with ages 16 years and over (thousands). This series is smoothed by eliminating breaks in population from 10-year censuses and post-2000 American Community Surveys using the "error of closure" method. This fairly simple method is used by the Census Bureau to get a smooth monthly population series and reduce the unusual influence of drastic demographic changes.[12]

CDX_USNA Consumer durable goods expenditures. Nominal personal consumption expenditures: durable goods (SAAR, millions of dollars).

nominveqipp Nominal equipment and intellectual property products investment (SAAR, millions of dollars).

AggHours Total hours in the nonfarm business (NFB) sector. It is calculated as (Average hours per workers in NFB sector) times (Total civilian employment from Household Survey). The series is normalized to one at 1948Q1.

A.3. *Estimation Procedure.* We apply the Bayesian methodology to the estimation of the log-linearized medium-scale structural model, using our own C/C++ code. The advantage of using our own code instead of using Dynare is the flexibility and accuracy we have for finding the posterior mode. We generate over a half million draws from the prior as a starting point for our optimization routine and select the estimated parameters that give the highest posterior probability density. The optimization routine is a combination of NPSOL software package and the csminwel routine provided by Christopher A. Sims.

In estimation, we use the log-linearized equilibrium conditions, reported in Supplemental Appendix E, to form the posterior probability function fit to the six quarterly U.S. time series from 1995Q2 to 2017Q2: the price–rent ratio in commercial real estate, the quality-adjusted relative price of investment, real per capita consumption, real per capita investment (in consumption units), and per capita hours worked. Excluding the four lags, the sample for estimation begins with 1996Q2 when the repeated-sales price of commercial real estate became available.

We fix the values of certain parameters as an effective way to sharpen the identification of some key parameters in the model. The capital share $\alpha(1 - \phi)$ is set at 0.33, consistent with the average capital income share. The share of land in production is estimated at $\phi = 0.07$ by solving the steady state (see Supplemental Appendix G). The growth rate of aggregate investment-specific technology, $g_z = 1.01$, is consistent with the average growth rate of the inverse relative price of investment goods. The growth rate of aggregate output, $g_\gamma = 1.003$, is consistent with the average common growth rate of consumption and investment. The interest rate R_f is set at 1.01. The steady-state capacity utilization u is set at 1. The steady-state labor

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[12] The detailed explanation can be found at http://www.census.gov/popest/archives/methodology/intercensal_nat_meth.html.

TABLE A.1
PRIOR DISTRIBUTIONS OF STRUCTURAL AND SHOCK PARAMETERS

Parameter	Distribution	a	b	Low	High
ν	Gamma(a,b)	1.0	3.0	0.017	1.000
χ	Gamma(a,b)	1.0	30	0.0017	0.100
δ''/δ'	Gamma(a,b)	4.6	17	0.100	0.500
γ	Beta(a,b)	1.0	2.0	0.026	0.776
Ω	Gamma(a,b)	1.0	0.5	0.100	6.000
ρ_z	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_{v_z}	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_a	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_{v_a}	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_θ	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_ξ	Beta(a,b)	1.0	5.0	0.010	0.450
ρ_ψ	Beta(a,b)	1.0	5.0	0.010	0.450
σ_z	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{v_z}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_a	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_{v_a}	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_θ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ξ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000
σ_ψ	Inv-Gam(a,b)	0.3261	1.45e04	0.0001	2.0000

NOTE: "Low" and "High" denote the bounds of the 90% probability interval for each parameter.

supply as a fraction of the total time is normalized at $N = 0.3$. To solve the steady state, we impose three additional restrictions to be consistent with the data: (i) the capital–output ratio is 1.125 at annual frequency; (ii) the investment–capital ratio is 0.22 at annual frequency; and (iii) the rental-income-to-output ratio is 0.1.[13]

We estimate five structural parameters as well as all the persistence and volatility parameters that govern exogenous shock processes. The five structural parameters are the inverse Frisch elasticity of labor supply ν, the collateral elasticity χ, the elasticity of capacity utilization $\delta''(1)/\delta'(1)$, the habit formation γ, and the investment adjustment cost Ω. The remaining parameters are then obtained from the steady-state relationships that satisfy the aforementioned data ratio restrictions. These parameters are: the capital depreciation rate ($\delta = 0.0437$), the subjective discount factor ($\beta = 0.993$), the collateral elasticity ($\chi = 0.045$), the capacity utilization rate ($\delta'(1) = 0.0638$), and the labor disutility ($\psi = 4.027$).

For the estimated parameters, we specify a prior that covers a wide range of values that are economically plausible (Table A.1). The prior for ν, χ, γ, or Ω has a distribution with the shape hyperparameter $a = 1$. This hyperparameter value is specified to allow a *positive* probability density at the zero value. The implied 90% prior probability bounds are consistent with the values considered in the literature. The prior distribution for $\delta''(1)/\delta'(1)$ is designed to cover the range consistent with Jaimovich and Rebelo (2009).

The prior for the persistence parameters of exogenous shock processes follows the beta distribution with the 90% probability interval between 0.01 and 0.45. Such a prior favors stationarity. The prior for the standard deviations of shock processes follows the inverse gamma distribution with the 90% probability interval between 0.0001 and 2.0. The standard deviation prior specification is far more diffuse than what is used in the literature.

[13] The output data used in our model are a sum of personal consumption expenditures and private domestic investment. Consumption is the private expenditures on nondurable goods and nonhousing services. Investment is the private expenditures on consumer durable goods and fixed investment in equipment and intellectual property. Accordingly, we measure capital stock using the annual stocks of equipment, intellectual products, and consumer durable goods.

A.4. *Endogenous Total Factor Productivity (TFP) and Its Relationship with the Survival Elasticity.* Log-linearizing the endogenous TFP

(A.14)
$$TFP_t = \frac{1}{1 - F(a_t^*)} \int_{a_t^*}^{\infty} af(a)da$$

yields

$$\widehat{TFP}_t = \frac{a^* f(a^*)}{1 - F(a^*)} \hat{a}_t^* - \frac{(a^*)^2 f(a^*)}{\int_{a^*}^{\infty} af(a)da} \hat{a}_t^*.$$

Define the survival elasticity

$$\eta = \frac{a^* f(a^*)}{1 - F(a^*)}.$$

From Equations (14) and (17), we deduce the wedge (due to the financial friction) as

$$\mu_t = \frac{Y_t}{P_{Xt}X_t} - 1 = \frac{\int_{a_t^*}^{\infty} af(a)da}{a_t^*(1 - F(a_t^*))} - 1.$$

The steady-state financial wedge is

$$\mu = \frac{\int_{a^*}^{\infty} af(a)da}{a^*(1 - F(a^*))} - 1.$$

With the definitions of η and μ, we have

$$\widehat{TFP}_t = \frac{a^* f(a^*)}{1 - F(a^*)} \frac{\mu}{1 + \mu} \hat{a}_t^* = \frac{\eta\mu}{1 + \mu} \hat{a}_t^*.$$

Log-linearizing the stationary version of Equation (16) gives us

(A.15)
$$\hat{a}_t^* = \frac{1 + \mu}{1 + \eta + \mu} \left(\hat{p}_t - \hat{Y}_t \right).$$

It follows that

$$\widehat{TFP}_t = \frac{\eta\mu}{1 + \eta + \mu} (\hat{p}_t - \hat{Y}_t).$$

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

Supporting Information S1
Supporting Information S2

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